Introduction to Probability
HW2 Solutions

(2.1) The sample space is the set of all strings ending in a "6". This can be schematized by associating 0 with any of $\{1,2,3,4,5\}$ and a 1 with $\{6\}$. Then the space is $\{1, 01, 001, 0001, \dots\}$. This includes the (infinite) string of all 0's (a "6" is never seen).

$E_n = \{$ all strings of at least $n-1$ 0's in a row with a 1 at the end $\}$: at least n rolls are required. In that case $\left(\bigcup_{n=1}^{\infty}\right)^c = \{000000\dots\}$, the string of all 0's.

(2.8) Have that $A \cap B = \phi$, $P(A) = .3, P(B) = .5$.
a) $P(A \cup B) = P(A) + P(B) = .8$
b) $P(A \cap B^c) = P(A) = .3$
c) $P(A \cap B) = P(\phi) = 0$

(2.17) There are $\binom{64}{8}$ ways to place 8 rooks on a chessboard. Now, certainly can only have one in each row. Then it comes down to choosing the columns: the rook in row one will have 8 choices of column, the rook in row two will have 7 choices of column, etc. Thus,

$$P\Big(\text{no rook can kill another}\Big) = \frac{8!}{\binom{64}{8}}.$$

(2.23) For the denominator, there are 36 possible outcomes. For the numerator count:
If die one = 1, second die has 5 choices.
If die one = 2, second die has 4 choices.
etc.
So: $Prob = \frac{5+4+3+2+1}{36} = \frac{15}{36}$.

(2.25) Following hint the probability desired is written as the sum of $P(E_n)$ for $n = 1$ to ∞ and

$$P(E_n) = \frac{\#\{n-1 \text{ ways to see neither 5 or 7, then 5}\}}{\#\{\text{outcomes of n rolls of two dice}\}}.$$

As there are 4 ways to see a five and, 6 ways to see a seven, $P(E_n) = (\frac{26}{36})^{n-1}(\frac{4}{36})$ and

$$\begin{aligned} P(5 \text{ before } 7) = \sum_{n=1}^{\infty} P(E_n) &= \frac{4}{36} \cdot \sum_{n=0}^{\infty} (\frac{26}{36})^n \\ &= \frac{4/36}{1 - 26/36} = 4/10. \end{aligned}$$

(2.32) There are $(b+g)!$ ways to line up $(b+g)$ different people. And:
$\#$ of ways to put a g in place i
$= g$ choices for that place
$\times\ (b+g-1)!$ ways to arrange the others.
Therefore, $P($ girl in place $i) = \frac{g \cdot (b+g-1)!}{(b+g)!} = \frac{g}{b+g}$.

(2.42) Turn the "at least" into its complement:

$$\begin{aligned} P(\text{ double 6 occurs at least once}) &= 1 - P(\text{ double 6 never occurs}) \\ &= 1 - \frac{35 \cdot 35 \cdots 35}{36 \cdot 36 \cdots 36} = 1 - \left(\frac{35}{36}\right)^n. \end{aligned}$$

(TH 2.18) As suggested

$$\begin{aligned} f_n &= \#\{n \text{ flips, no } HH\} \\ &= \#\{\text{ditto, first toss is } T\} + \#\{\text{ditto, first toss is } H\}. \end{aligned}$$

If first toss is a T, left to count the number of ways that there is no HH in a series of $n-1$ coin tosses. But this is f_{n-1} bt definition.

If first toss is a H, then the second must be a T and everything "starts over". Now need no HH is $n-2$ tosses, that is f_{n-2}.

This proves the identity. Dividing through by 2^n you find $p_n = (1/2)p_{n-1} + (1/4)p_{n-2}$. p_{10} is then found by iteration.

(TH 2.20) Cannot have a countably infinite sample space of equally likely outcomes. Would be forced to conclude $P(\omega_n) = 0$ for each sample point ω_n and so also $P(\Omega) = \sum_{n=1}^{\infty} P(\omega_n) = 0$ which violates the axioms.

If we allow $P(\omega_n) = 2^{-n}$ for $\{\omega_1, \omega_2, \dots\}$ we have a perfectly good Prob space with a countably infinite sample space.