Introduction to Probability
HW1 Solutions

(1.4) Unrestricted there are $4\cdot 3\cdot 2\cdot 1 = 4! = 24$ possibilities. If the first two slots are restricted then the count is $2\cdot 1\cdot 2\cdot 1 = 4$.

(1.8) (a) $5!$
(b) $7!/(2!\,2!)$ (dividing out the overcounts from PP and OO).
(c) 11 letters, 4 S's, 4 I's, and 2 P's. So the count is $\frac{11!}{4!\,4!\,2!}$.
(d) $7!/(2!\,2!)$

(1.13) This is the same as asking how many distinct pairs exist in a group of 20. That is $\binom{20}{2}$.

(1.19) (a) Two cases. If neither is on $= \binom{8}{3}\binom{4}{3}$. If one is on then $= \binom{2}{1}\binom{8}{3}\binom{4}{2}$ in which the first factor counts which one will serve. Add these to cases for total.
(b) This is similar: $\binom{6}{3}\binom{4}{3} + \binom{2}{1}\binom{6}{2}\binom{4}{3}$.
(c) Focus on the man. If he serves $= \binom{1}{1}\binom{7}{3}\binom{5}{2}$ (this follows as: pick him, then three of seven women who *will* serve with him, then two more men). If he does not serve then $= \binom{5}{3}\binom{8}{3}$. Add for total.

(1.21) Same as making a word of length 7 with 4 R's ("rights") and 3 U's ("ups"). Thus $= \binom{7}{4}$.

(1.22) Now want a word of length 4 using $\{R, R, U, U\}$ and another of length 3 from $\{R, R, U\}$. Thus have $= \binom{4}{2}\binom{3}{2}$.

(1.31) A thinly disguised partition problem. If we allow a school to go empty handed (allowing 0's in the partition) then have $\binom{8+4-1}{4-1} = \binom{11}{3}$. If each school must get at least one then have $\binom{8-1}{4-1} = \binom{7}{3}$.

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(Th 8) To verify

$$\binom{n+m}{r} = \sum_{k=0}^{r} \binom{n}{k}\binom{m}{r-k}$$

note that the left counts the number of groups of size r from a larger set of size $n+m$. Imagine splitting the larger set into two subgroups of size n and m. Then a group of size r of the big set can be made by first picking k from the n-group ($k \le r$) then the rest from the m-group. If you do this for all appropriate k, you finish the job of enumerating all r groups of the big set.

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(Th 11) Want to show that

$$\binom{n}{k} = \binom{k-1}{k-1} + \binom{k}{k-1} + \cdots \binom{n-1}{k-1}.$$

Imagine numbering the members from 1 to n. You can choose k such that:

Member k is the biggest in $1 = \binom{k-1}{k-1}$ ways,
Member $k+1$ is the biggest in $\binom{k}{k-1}$ ways,
Member $k+2$ is the biggest in $\binom{k+1}{k-1}$ ways,
etc.

Now each of these subcounts is disjoint (yields distinct group of k). All together you produce all poosible.