MATH 2400: CALCULUS 3

MAY 9, 2007

FINAL EXAM

I have neither given nor received aid on this exam.

Name:____

- ◯ **001** E. Kim .... (9AM)
- ◯ **002** E. Angel ....(10AM)
- ◯ **003** I. Mishev .... (11AM)
- ◯ **004** M. Daniel .... (12AM)
- ◯ **005** A. Gorokhovsky ...... (1PM)

If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show all of your work, and give adequate explanations.

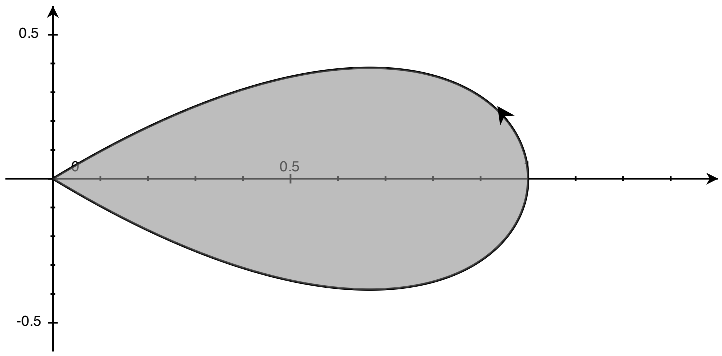
DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	15 pts	
2	15 pts	
3	15 pts	
4	15 pts	
5	20 pts	
6	15 pts	
7	30 pts	
8	15 pts	
9	15 pts	
10	30 pts	
11	15 pts	
TOTAL	200 pts	

1. (15 pt) Find the area of the region enclosed by the curve

$$x = 1 - t^2,\ y = t(1 - t^2),\ -1 \le t \le 1$$

(see the picture below)



2. (15 pt) Find the equation for the plane tangent to the paraboloid $z = 2x^2 + 3y^2$ that is also parallel to the plane $4x - 3y - z = 10$

3. (15 pt) Find the flux of $\mathbf{F}(x, y, z) = (x + y)\mathbf{i} + (y + z)\mathbf{j} + (z + x)\mathbf{k}$ across the portion of the plane $x + y + z = 1$ in the first octant oriented by unit normals with positive components.

4. (15 pt) Evaluate the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \cos\left(y^3\right) dy\, dx$$

by changing the order of integration.

5. A wire 12 cm long is cut into three or fewer pieces, with each piece bent into a square.

(a) (10 pt) What is the minimal total area of the squares? Justify your answer.

(b) (10 pt) What is the maximal total area of the squares? Justify your answer.

6. (15 pt) Evaluate $\oint_C \mathbf{F} \bullet d\mathbf{r}$, where $\mathbf{F}(x, y, z) = -3y^2\mathbf{i} + 4z\mathbf{j} + 6x\mathbf{k}$ and C is the triangle in the plane $z = \frac{1}{2}y$ with vertices $(2, 0, 0)$, $(0, 2, 1)$ and $(0, 0, 0)$ with a counterclockwise orinetation looking down the positive z-axis.

7. The helix $\mathbf{r}_1(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ intersects the curve $\mathbf{r}_2(t) = (1+t)\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ at the point $P(1, 0, 0)$.

(a) (15 pt) Find the tangent line to each curve at P.

(b) (15 pt) Find the angle between the tangent lines at P.

8. (15 pt) Evaluate $\iint\limits_R \sin\left(\frac{1}{2}(x+y)\right)\cos\left(\frac{1}{2}(x-y)\right)dA$, where R is the triangular region whose vertices are $(0,0)$, $(2,0)$ and $(1,1)$.

9. (15 pt) Consider $\int_{(0,0)}^{(3,2)}(2xe^y+1)dx + x^2e^y dy$. Show that the integral is independent of path and evaluate the integral using the Fundamental Theorem of Line Integrals.

10. Find

(a) (15 pt) the equation of the plane through $P(1,-1,2)$, $Q(2,1,3)$, and $R(-1,2,-1)$.

(b) (15 pt) parametric equations of the line of intersection of the planes

$$x + 2y + z = 1 \text{ and } x - y + 2z = -8.$$

11. (a) (1 pt) Write a formula expressing the directional derivative $D_{\mathbf{n}}f$ of a function f along a unit vector $\mathbf{n}$ in terms of ∇f and $\mathbf{n}$.

(b) (14 pt) Let σ be the sphere $x^2 + y^2 + z^2 = 1$, let $\mathbf{n}$ be an outward unit normal, and let $D_{\mathbf{n}}f$ be the directional derivative of $f(x, y, z) = x^2 + e^x \cos y + y^2 + z^2$. Evaluate $\iint_{\sigma} D_{\mathbf{n}} f \, dS$.