

Complex Analysis: Homework 1

September 5, 2008

Ahlfors §2.1.2 1-6.

Rudin 11.1 Suppose u and v are real harmonic functions in a plane region Ω . Under what conditions is uv harmonic? Show that u^2 cannot be harmonic in Ω unless u is constant. For which functions f , analytic on Ω is $|f|^2$ harmonic?

Rudin 11.2 Suppose f is a complex function in a region Ω , and both f and f^2 are harmonic in Ω . Prove that either f or \bar{f} is holomorphic in Ω .

Rudin 11.3 If u is a harmonic function in a region Ω , what can you say about the set of points at which the gradient of u is 0?