

Introduction to Probability: Homework 1

July 9, 2008

- 1.4 John, Jim, Jay and Jack have formed a band consisting of 4 instruments. If each can play all 4 instruments, how many different arrangements are possible? What if John and Jim can play all 4 instruments, but Jay and Jack can each only play piano and drum?
- 1.8 How many different letter arrangements can be made from the letters
- (a) FLUKE;
 - (b) PROPOSE;
 - (c) MISSISSIPPI;
 - (d) ARRANGE?
- 1.13 Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?
- 1.19 From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many committees are possible if:
- (a) 2 of the men refuse to serve together;
 - (b) 2 of the women refuse to serve together;
 - (c) 1 man and 1 woman refuse to serve together.
- 1.21
- 1.22
- 1.31 If 8 identical blackboards are to be divided among 4 schools, how many divisions are possible? How many, if each school must receive 1 blackboard?

Th 1.8 Prove that

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}$$

Hint: Consider a group of n men and m women. How many groups of size r are possible?

Th 1.11 The following identity is known as Fermat's combinatorial identity.

$$\binom{n}{k} = \sum_{i=k}^n \binom{i-1}{k-1} \quad n \geq k.$$

Give a combinatorial argument (no computations are needed) to establish this identity. Hint: Consider the set of numbers 1 through n . How many subsets of size k have i as their highest-numbered member?