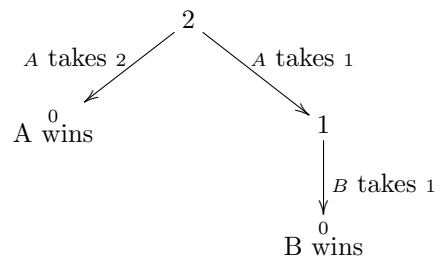


Game theory: Taking one or two

Play this game with a partner. To set up the board, place any number of chips in a row (the arrangement doesn't matter, just the number of chips). Players take turns. On your turn, you may remove either 1 or 2 chips from the row (it doesn't matter which ones; all chips are identical). The last player to take chips (leaving an empty board) wins.

1. Play the game until you understand the rules.
2. Each player has two choices on his/her turn. Therefore, we can draw a *game tree* showing all the possible ways the game may play out. For example, the game tree for a game of 2 chips looks like this:



If the first player plays intelligently for the game of 2 chips, can he/she always win?

3. Draw the game tree for 3 chips. Can the first player always win?

4. Draw the game tree for 4 chips. Can the first player always win?

5. Draw the game tree for 5 chips. Can the first player always win?

6. For what starting number of chips can the first player always win? Try drawing a chart and playing some test games to check your guess.

7. Explain the winning strategy for playing this game. Play some games to test this strategy.

8. Describe some generalizations of this game. (For example, more than 2 players, different kinds of chips, ability to take more than 2 chips per turn, etc.)

9. What are some other games for which game trees can be used?