## Homework \#3

1. Let

$$
F(x)= \begin{cases}x^{2} \sin \left(x^{-2}\right), & \text { if } x \neq 0 \\ 0, & \text { if } x=0\end{cases}
$$

Show that $F$ is differentiable everywhere (including at $x=0$ ), but $F \notin B V([-1,1])$.
2. Give an example of an increasing function $F: \mathbb{R} \rightarrow \mathbb{R}$ whose set of discontinuities is $\mathbb{Q}$.
3. If $F: \mathbb{R} \rightarrow \mathbb{C}$ is Lipschitz continuous, show that $F$ is differentiable almost everywhere. Give an example to show that $F$ may not be differentiable everywhere.
4. If $F: \mathbb{R} \rightarrow \mathbb{R}$ is increasing, show that $F(b)-F(a) \geq \int_{a}^{b} F^{\prime}(t) d t$ for every $a<b$. Give an example to show that the inequality may be strict.

