## Homework #3

1. Let

$$F(x) = \begin{cases} x^2 \sin(x^{-2}), & \text{if } x \neq 0\\ 0, & \text{if } x = 0. \end{cases}$$

Show that F is differentiable everywhere (including at x = 0), but  $F \notin BV([-1, 1])$ .

2. Give an example of an increasing function  $F : \mathbb{R} \to \mathbb{R}$  whose set of discontinuities is  $\mathbb{Q}$ .

- 3. If  $F : \mathbb{R} \to \mathbb{C}$  is Lipschitz continuous, show that F is differentiable almost everywhere. Give an example to show that F may not be differentiable everywhere.
- 4. If  $F : \mathbb{R} \to \mathbb{R}$  is increasing, show that  $F(b) F(a) \ge \int_a^b F'(t) dt$  for every a < b. Give an example to show that the inequality may be strict.