Practice Final

The following is a list of problems I consider final-worthy. This list of problems should serve as a good place to start studying, and it should not be considered a comprehensive list of problems from the sections we've covered. YOU are responsible for studying all the sections to be covered on the final.

Note: These problems concentrate on material since the last exam. To review earlier material, use the review problems distributed for the first two exams, as well as the exams themselves.

1. A single observation of a random variable having a uniform density with $\alpha = 0$ is used to test the null hypothesis $\beta = \beta_0$ against the alternative hypothesis $\beta = \beta_0 + 5$. If the null hypothesis is rejected if and only if the random variable takes on a value greater than $\beta_0 + 1$, find the probabilities of type I and type II errors.
2. For large n, the sampling distribution of S is sometimes approximated with a normal distribution having mean σ and variance $\frac{\sigma^2}{2n}$. Show that this approximation leads to the following $(1-\alpha)100\%$ confidence interval for σ:
$$\frac{s}{1 + \frac{z_{\alpha/2}}{\sqrt{2n}}} < \sigma < \frac{s}{1 - \frac{z_{\alpha/2}}{\sqrt{2n}}}.$$
3. A random sample of size n from an exponential population is used to test the null hypothesis $\theta = \theta_0$ against the alternative hypothesis $\theta = \theta_1 < \theta_0$. Use the Neyman–Pearson lemma to find the form of a most powerful critical region.
4. A random sample of size n from an exponential population is used to test the null hypothesis $\theta = \theta_0$ against the alternative hypothesis $\theta \neq \theta_0$. Find the corresponding critical region using the likelihood ratio test.
5. Given a random sample of size n from a uniform population with $\alpha = 3$, use the method of moments to obtain a formula for estimating the parameter β.
6. The data in Example 16.5 represents the weights in pounds both before and after a diet for a group of 16 individuals. We will only consider the weights after the diet. Use the sign test to test at the 0.05 level of significance whether the mean weight after the diet is 180 pounds against the alternative that it is higher than 180 pounds.
7. Suppose that it is known from experience that the standard deviation of the weight of 12-ounce packages of cookies is 0.25 ounces. We wish to check whether the true average weight of the packages is 12 ounces. 25 packages are selected at random and their mean is found to be 12.075. Test the null hypothesis $\mu = 12$ against the alternative $\mu \neq 12$ at the 0.01 level of significance by using a P-value, assuming the distribution of weights is normal.
8. If $x = 6$ of $n = 20$ patients suffered serious side effects from a new medication, test the null hypothesis $\theta = 0.40$ against the alternative $\theta \neq 0.40$ at the 0.05 level of significance. Here, θ represents the true proportion of patients suffering serious side effects from the new medication.
9. In a random sample, 147 out of 400 persons given a flu vaccine went on to develop the flu. Construct a 95% confidence interval for the true proportion of vaccinated individuals who will develop the flu.
10. The data in Example 16.3 represents the number of accidents that occurred at 12 dangerous intersections during four weeks before and four weeks after the installation of a new traffic-control system. Use the paired-sample signed-rank test at the 0.05 level of significance to test the null hypothesis that the new traffic-control system is only as effective as the old system.
11. Show that $\bar{X}$ is a sufficient estimator of the mean μ of a normal population with known variance σ^2.

Select solutions and hints:

2. Since S is approximately normally distribution with mean σ and standard deviation $\sigma/\sqrt{2n}$, it follows that
$$\mathbb{P}\left(\left|\frac{S-\sigma}{\sigma/\sqrt{2n}}\right| \leq z_{\alpha/2}\right) \approx 1-\alpha.$$
In order to complete the problem we will need to solve for σ in the following inequality:
$$\left|\frac{s-\sigma}{\sigma/\sqrt{2n}}\right| \leq z_{\alpha/2}.$$
Indeed, this inequality is equivalent to
$$-z_{\alpha/2}\frac{\sigma}{\sqrt{2n}} \leq s-\sigma \leq z_{\alpha/2}\frac{\sigma}{\sqrt{2n}}.$$
Solving for σ we conclude that
$$\frac{s}{1+\frac{z_{\alpha/2}}{\sqrt{2n}}} < \sigma < \frac{s}{1-\frac{z_{\alpha/2}}{\sqrt{2n}}}$$
is an approximate $(1-\alpha)100\%$ confidence interval for σ.

5. The method of moments gives the approximation
$$\overline{x} = \frac{3+\beta}{2},$$
so we find the estimator $\hat{\beta} = 2\overline{x} - 3$.

9. Since n is large, we use the standard normal approximation to the binomial with $\hat{\theta} = \frac{147}{400} \approx 0.37$. Using $z_{\alpha/2} = 1.96$, we obtain the 95% confidence interval $0.32 < \theta < 0.42$.

11. We worked out the details for this problem as an example in class.