Practice Midterm

The following is a list of problems I consider midterm-worthy. This list of problems should serve as a good place to start studying, and it should not be considered a comprehensive list of problems from the sections we've covered. YOU are responsible for studying all the sections to be covered on the midterm.

1. Let $X_1, \ldots, X_n$ be a random sample of size n from a uniform population on $[0, \beta]$ (i.e. $\alpha = 0$). Show that $X_{(n)}$ is a consistent estimator of β.
2. A single observation of a random variable having a uniform density with $\alpha = 0$ is used to test the null hypothesis $\beta = \beta_0$ against the alternative hypothesis $\beta = \beta_0 + 2$. If the null hypothesis is rejected if and only if the random variable takes on a value greater than $\beta_0 + 1$, find the probabilities of type I and type II errors.
3. To estimate the average time required for certain repairs, an automobile manufacturer had 40 mechanics, a random sample, timed in the performance of this task. If it took them on the average 24.05 minutes with a standard deviation of 2.68 minutes, what can the manufacturer assert with 95% confidence about the maximum error if he uses $\overline{x} = 24.05$ minutes as an estimate of the actual mean time required to perform the repairs.
4. Let X be a binomial random variable with parameters (n, θ). Show that $\frac{X+1}{n+1}$ is a biased estimator of the parameter θ. Is this estimator asymptotically unbiased?
5. Given a random sample of size n from a gamma population, use the method of moments to obtain formulas for estimating the parameters α and β.
6. Given a random sample of size n from a gamma population with known parameter α, find the maximum likelihood estimator for β.
7. For large n, the sampling distribution of S is sometimes approximated with a normal distribution having mean σ and variance $\frac{\sigma^2}{2n}$. Show that this approximation leads to the following $(1-\alpha)100\%$ confidence interval for σ:
$$\frac{s}{1 + \frac{z_{\alpha/2}}{\sqrt{2n}}} < \sigma < \frac{s}{1 - \frac{z_{\alpha/2}}{\sqrt{2n}}}.$$
8. A random sample of size n from an exponential population is used to test the null hypothesis $\theta = \theta_0$ against the alternative hypothesis $\theta = \theta_1 > \theta_0$. Use the Neyman–Pearson lemma to find the most powerful critical region of size α.
9. Given a random sample of size n from a uniform population with $\alpha = 3$, use the method of moments to obtain a formula for estimating the parameter β.