

Practice Midterm

The following is a list of problems I consider midterm-worthy. This list of problems should serve as a good place to start studying, and it should not be considered a comprehensive list of problems from the sections we've covered. YOU are responsible for studying all the sections to be covered on the midterm.

1. A random sample of size $n = 225$ is to be taken from an infinite exponential population with $\theta = 4$. Based on the central limit theorem, what is the probability that the mean of the sample will exceed 4.5?
2. Poker dice is played by simultaneously rolling 5 dice. Show that
 - (a) $\mathbb{P}(\text{no two alike}) = 0.0926$.
 - (b) $\mathbb{P}(\text{one pair}) = 0.4630$.
 - (c) $\mathbb{P}(\text{two pair}) = 0.2315$.
 - (d) $\mathbb{P}(\text{three alike}) = 0.1543$.
 - (e) $\mathbb{P}(\text{five alike}) = 0.0008$.
3. Let X_1, \dots, X_n be a random sample of size n from a continuous uniform population on the interval $[0, 1]$ (i.e. $\alpha = 0, \beta = 1$). Find the expectation and variance for the order statistic $X_{(n)}$.
4. An infinite sequence of independent trials is to be performed. Each trial results in a success with probability p and a failure with probability $1 - p$. What is the probability that
 - (a) at least k successes occur in the first n trials?
 - (b) all trials result in successes?
5. A random sample of size $n = 100$ is taken from an infinite population with mean $\mu = 40$ and variance $\sigma^2 = 9$. Use Chebyshev's inequality to estimate the probability that the value we obtain for \bar{X} is between 37 and 43?
6. Roll two dice. What is the probability that the sum of the dice is 7 given that the product is 12?
7. If S_1^2 and S_2^2 are the variances of independent random samples of sizes $n_1 = 10$ and $n_2 = 15$ from a normal population with $\sigma_1^2 = 12$ and $\sigma_2^2 = 18$, find $\mathbb{P}(S_1^2/S_2^2 > 1.16)$.
8. Let X be a random variable which takes the values $-1, 0, 1$ with equal probability. Compute the moment generating function of X and use it to find the variance of X .
9. How many different samples of size $n = 10$ can be drawn from a finite population of size $N = 36$? What is the probability of each of these samples?