MTH 202 REFERENCE

Formulas

Derivative Rules

Constant	$\frac{d}{dx}c = 0$
	$\frac{d}{dx}x = 1$
Constant Multiple	$\frac{d}{dx}cx = c$
Power Rule	$\frac{d}{dx}(x^r) = rx^{r-1}$
Exponential	$\frac{d}{dx}(a^x) = a^x \ln(a)$
	$\frac{d}{dx}(e^x) = e^x$
Log	$\frac{d}{dx}\log_a x = \frac{1}{x\ln(a)}$
	$\frac{d}{dx}\ln x = \frac{1}{x}$
Product Rule	$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + g'(x)f(x)$
Quotient Rule	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x)-g'(x)f(x)}{(g(x))^2}$
Chain Rule	$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$
Trig Derivatives	$\frac{d}{dx}\sin x = \cos x$
	$\frac{d}{dx}\cos x = -\sin x$
	$\frac{d}{dx}\tan x = \sec^2 x$
	$\frac{d}{dx}\csc x = -\csc x \cot x$
	$\frac{d}{dx}\sec x = \sec x \tan x$
	$\frac{d}{dx}\cot x = -\csc^2 x$
Inverse Trig Derivatives	$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$
	$\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}$
	$\frac{d}{dx}\arctan x = \frac{1}{1-x^2}$

Trig Identities

Pythagorean Identities	$\sin^2 x + \cos^2 x = 1$
	$1 - \sin^2 x = \cos^2 x$
	$1 - \cos^2 x = \sin^2 x$
	$\tan^2 x = \sec^2 x - 1$
	$\cot^2 x = \csc^2 x - 1$
Sum and Difference Identities	$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$
	$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$
Double Angle Identities	$\sin 2x = 2\sin x \cos x$
	$\cos 2x = \cos^2 x - \sin^2 x$
	$\sin x \cos x = \frac{1}{2}\sin 2x$
Half Angle Identities	$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
	$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
	$\sin a \cos b = \frac{1}{2}[\sin(a-b) + \sin(a+b)]$
	$\sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$
	$\cos a \cos b = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$

Antiderivative Rules

f and g are continuous functions.

Constant	$\int k\,dx = kx + C$
Power Rule	$\int x^n\,dx = \frac{x^n+1}{n+1} + C, (n \neq -1)$
	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
Exponential	$\int a^x\,dx = \frac{a^x}{\ln a} + C$
	$\int e^x\,dx = e^x + C$
Substitution Rule	$\int_a^b f(g(t))g'(t)\,dt = \int_{g(a)}^{g(b)} f(x)\,dx,\ (g'(t) \neq 0, t \in [a,b])$
Integration by Parts	$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$

Trig Antiderivatives	$\int \sin x\,dx = -\cos x + C$
	$\int \cos x\,dx = \sin x + C$
	$\int \sec^2 x\,dx = \tan x + C$
	$\int \csc x \cot x\,dx = -\csc x + C$
	$\int \sec x \tan x\,dx = \sec x + C$
	$\int \csc^2 x\,dx = -\cot x + C$
	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$
	$\int \tan x\,dx = \ln\lvert\sec x\rvert + C$
	$\int \csc x\,dx = -\ln\lvert\csc x + \cot x\rvert + C$
	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$
Inverse Trig Antiderivatives	$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$
	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C$
	$\int \sqrt{1+x^2}\,dx = \frac{1}{2}\left(x\sqrt{1+x^2} + \ln\lvert\sqrt{1+x^2}+x\rvert\right) + C$

Other Integration Formulas

Reduction Formulas	$\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
	$\int \cos^n x\,dx = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
Volume by disks (around x-axis)	$V = \pi \int (r(x))^2\,dx$
Volume by washers (around x-axis)	$V = \pi \int (r_o(x))^2 - (r_i(x))^2\,dx$
Volume by shells (around y-axis)	$V = 2\pi \int r(x) f(x)\,dx$
Arc Length	$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$
	$L = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$
Surface Area	$A = \int 2\pi y\,ds$
	$A = \int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$
	$A = \int_c^d 2\pi y\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$

Series

Common Series	$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
	$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$
	$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

Harmonic Series.

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges.}$$

P Series.

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{diverges if } p \leq 1 \\ \text{converges if } p > 1 \end{cases}$$

Geometric Series.

$$\sum_{n=1}^{\infty} r^n \begin{cases} \text{diverges if } |r| \geq 1 \\ \text{converges if } |r| < 1 \end{cases}$$

Comparison Test. Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms.

(1) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.
(2) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.

Limit Comparison Test. Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If

$$\lim_{a \to \infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$, then either both series converge or both series diverge.

Integral Test. Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)\, dx$ is convergent.

Alternating Series Test. If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - \dots \quad b_n > 0$$

satisfies

(1) $b_{n+1} \leq b_n$ for all n
(2) $\lim_{n \to \infty} b_n = 0$

then the series is convergent.

Absolute Convergence Test. If $\sum |a_n|$ converges, $\sum a_n$ converges.

Ratio Test.

(1) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

(2) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series diverges.

(3) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, then the Ratio Test is inconclusive.

Root Test.

(1) If $\lim_{n\to\infty}\sqrt[n]{|a_n|} = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

(2) If $\lim_{n\to\infty}\sqrt[n]{|a_n|} = L > 1$ or $\lim_{n\to\infty}\sqrt[n]{|a_n|} = \infty$, then the series diverges.

(3) If $\lim_{n\to\infty}\sqrt[n]{|a_n|} = 1$, then the Ratio Test is inconclusive.

Parametric Equations

Let $x = f(t)$ and $y = g(t)$.

Derivative	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$
Integral	$\int_a^b g(t)f'(t)dt$
Arc Length	$L = \int_\alpha^\beta \sqrt{(f'(t))^2 + (g'(t))^2}\, dt$

Polar Equations

Derivative	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$
Integral	$\int_a^b \frac{1}{2}(r(\theta))^2\, d\theta$