MTH 201 REFERENCE

Named Theorems

Squeeze Theorem/Sandwhich Theorem:

Suppose that for all x near a, $f(x) \geq g(x) \geq h(x)$
and
$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L.$$
Then:
$$\lim_{x\to a} g(x) = L$$

Intermediate Value Theorem (IVT):

If f is continuous on a closed interval $[a, b]$
and if N is a number between $f(a)$ and $f(b)$,
then there exists a number c between a and b such that $f(c) = N$.

Extreme Value Theorem (EVT):

If f is continuous on a closed interval $[a, b]$,
then f attains an absolute maximum $(c, f(c))$
and an absolute minimum $(d, f(d))$
at some c and d in $[a, b]$.

Fermat's Theorem:

If f has a local maximum or minimum at $(c, f(c))$
and $f'(c)$ exists,
then $f'(c) = 0$.

Mean Value Theorem for Derivatives:

If $f(x)$ is continuous on the closed interval $[a, b]$
and $f(x)$ is differentiable on the open interval (a, b),
then there exists some number c in the interval (a, b) where
$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Rolle's Theorem:

If $f(x)$ is continuous on the closed interval $[a, b]$
and $f(x)$ is differentiable on the open interval (a, b)
and $f(a) = f(b)$,
then there exists some number c in the interval (a, b) where $f'(c) = 0$

Racetrack Principle:

If $f(x)$ and $g(x)$ are continuous, differentiable functions on the closed interval $[a, b]$,
and $f'(x) \geq g'(x)$ for all x in $[a, b]$,
and $f(a) \geq g(a)$,
then $f(b) \geq g(b)$.

L'Hôpital's Rule (L'H):

Suppose f and g are differentiable and $g'(x) \neq 0$ for x near a
and
$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = 0$$
or
$$\lim_{x\to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x\to a} g(x) = \pm\infty$$

then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

provided the limits exist.

Fundamental Theorem of Calculus (FTC), Part 1:

If f is continuous on $[a, b]$,
and the function $g(x)$ is defined as

$$g(x) = \int_a^x f(t)\, dt,$$

where x is between a and b,
then $g(x)$ is continuous on $[a, b]$ and differentiable on (a, b)
and $g'(x) = f(x)$

Fundamental Theorem of Calculus (FTC), Part 2:

If f is continuous on the interval $[a, b]$,
then

$$\int_a^b f(t)\, dt = F(b) - F(a),$$

where F is any antiderivative of f.

Net Change Theorem:

The integral of a rate of change is a net change.

$$\Delta f(x) = \int_a^b f'(x)\, dx.$$

Important Definitions

continuity: A function f is continuous at point $x = a$ if the following conditions hold:

(1) $f(a)$ exists; a is in the domain of f
(2) $\lim_{x\to a} f(x)$ exists and is finite.
(3) $\lim_{x\to a} f(x) = f(a)$.

derivative: The derivative of a function f at a point $x = a$, denoted $f'(a)$, is given by

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} = \lim_{h\to 0} \frac{f(a + h) - f(a)}{h}.$$

differentiability: If the limit from the definition above exists, f is differentiable at a.

smooth: A function is smooth if you may take its derivative infinitely many times.

absolute/global max/min: The highest or lowest point attained by a function within a domain. There is an absolute maximum at $(c, f(c))$ if $f(c) \geq f(x)$ for all x in the domain of f.

local max/min: The highest or lowest point in nearby area. There is a local maximum at $(c, f(c))$ if $f(c) \geq f(x)$ for all x near c.

indeterminate forms: Forms of limits for which you cannot tell what the answer is without more work. They include:

(1) "$\frac{0}{0}$"
(2) "$\pm\frac{\infty}{\infty}$"

(3) "$0 \times \pm\infty$"
(4) "$\infty - \infty$"
(5) "0^0"
(6) "∞^0"
(7) "1^∞"

concavity: Concave up or concave down. f is concave up on an interval if $f''(x) > 0$ and concave down if $f''(x) < 0$ for all x in that interval.

point of inflection: $(c, f(c))$ is a point of inflection of f if $f''(x)$ is of opposite signs immediately above and below c.

antiderivatives: A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I. The most general antiderivative of f may be found by adding a constant C to any antiderivative of f.

differential equation: An equation that relates a function and its derivatives.

definite integral: Let f be a continuous function on a closed interval $[a, b]$, divide $[a, b]$ into n equal length subintervals ($\Delta x = \frac{b-a}{n}$) and label the endpoints of those subintervals $x_0 = a, x_1 = a + \Delta x, \ldots, x_n = b$. Let x_i^* be designated points in those subintervals ($x_i \leq x_{i+1}^* \leq x_{i+1}$). The definite integral of f on $[a, b]$ is:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

indefinite integral: Given a function f which is continuous on some interval, the indefinite integral of f is $\int f(x)\,dx = F(x) + C$ where $F(x)$ is an antiderivative of $f(x)$.

Formulas

a, c, k, r, etc. are constants.

Derivative Rules.

$$\frac{d}{dx}c = 0$$

$$\frac{d}{dx}x = 1$$

$$\frac{d}{dx}cx = c$$

Power Rule:

$$\frac{d}{dx}(x^r) = rx^{r-1}$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}\log_a x = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

Product Rule:

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + g'(x)f(x)$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$$

Trig Derivatives:

$$\frac{d}{dx}\sin x = \cos x$$
$$\frac{d}{dx}\cos x = -\sin x$$
$$\frac{d}{dx}\tan x = \sec^2 x$$
$$\frac{d}{dx}\csc x = -\csc x \cot x$$
$$\frac{d}{dx}\sec x = \sec x \tan x$$
$$\frac{d}{dx}\cot x = -\csc^2 x$$

Chain Rule:

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Inverse Trig Derivatives:

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

Linear Approximation.

$$L_f(x) = f'(a)(x-a) + f(a)$$

Trig Identities.

Pythagorean Identities:

$$\sin^2 x + \cos^2 x = 1$$
$$\sin^2 x - 1 = \cos^2 x$$
$$\cos^2 x - 1 = \sin^2 x$$
$$\tan^2 x = \sec^2 x - 1$$
$$\cot^2 x = \csc^2 x - 1$$

Double Angle Identities:

$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$

Properties of Definite Integrals.

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b c\,dx = c(b-a)$$

$$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\text{if } f(x) > 0 \text{ for all } x \text{ in } [a,b]\text{, then } \int_a^b f(x)\,dx > 0$$

$$\text{if } f(x) > g(x) \text{ for all } x \text{ in } [a,b]\text{, then } \int_a^b f(x)\,dx > \int_a^b g(x)\,dx$$

$$\text{if } m \le f(x) \le M \text{ for all } x \text{ in } [a,b]\text{, then } m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

Antiderivative Rules.

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^n+1}{n+1} + C, (n \neq 1)$$

$$\int e^x\,dx = e^x + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$