Midterm II

Intro to Discrete Math

MATH 2001

Spring 2022

Friday March 18, 2022

NAME: ______________________________

PRACTICE EXAM

Question:	**1**	**2**	**3**	**4**	**5**	Total
Points:	20	20	20	20	20	100
Score:						

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with the questions in the correct order, etc.
- You have 45 minutes to complete the exam. **We will spend the last 5 minutes of class to upload your exam to Canvas.**

1. (20 points) • **TRUE** or **FALSE**:

$$\textit{If } n \in \mathbb{N}, \textit{ then } \binom{2n}{n} \textit{ is even.}$$

If true, give a *direct proof* of the statement. If false, provide a *counter example*, and prove that it is a counter example. Your solution must start with the sentence, "*This statement is TRUE*," or the sentence, "*This statement is FALSE*."

1
20 points

2. (20 points) • In class we showed that the equation $x^2 + y^2 = 3$ has no rational solutions. Use this fact to give a *proof by contradiction* of the statement:

If k is an odd positive integer, then the equation $x^2 + y^2 = 3^k$ *has no rational solutions.*

2
20 points

3. (20 points) • For all real numbers $a, b \in \mathbb{R}$, give a *proof by induction* that for each natural number n the following statement is true:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}.$$

You may use, *without proof*, the fact that $\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$.

3
20 points

4. (20 points) • Suppose R is an equivalence relation on a set A, with four equivalence classes. *How many different equivalence relations S on A are there for which $R \subseteq S$?* You must prove that your answer is correct.

4
20 points

5. • **TRUE** or **FALSE**. For this problem, and this problem only, **you do <u>not</u> need to justify your answer**.

(a) (4 points) **TRUE** or **FALSE** (circle one). The LaTeX code

`x^100+3\pi x^2+5`

produces the following:

$$x^100 + 3\pi x^2 + 5$$

(b) (4 points) **TRUE** or **FALSE** (circle one). If R and S are equivalence relations on a set A, then $R \cap S$ is also an equivalence relation on A.

(c) (4 points) **TRUE** or **FALSE** (circle one). The empty set defines a reflexive relation on any set.

(d) (4 points) **TRUE** or **FALSE** (circle one). If $\sim$ is an equivalence relation on a set A and $a \in A$, then the equivalence class of a is the set $[a] = \{x \in A : \exists y \in A, x \sim y\}$.

(e) (4 points) **TRUE** or **FALSE** (circle one). If $\sim$ is an equivalence relation on a set A then the set of equivalence classes $A/\sim$ is a partition of the set A.

5
20 points