

HOMework 1

MATH 2001

SEBASTIAN CASALAINA

ABSTRACT. This is the first homework assignment. The problems are from Hammack [[Ham13](#), Ch. 1, §1.1–2]:

- **Section 1.1**, Exercises: 8, 18, 40.
- **Section 1.2**, Exercises: 2, 4, 12.

I worked with the entire class on **Section 1.1**, Exercises: 8, 18, 40.

CONTENTS

Chapter 1 Section 1.1	2
Ch.1, §1.1, Exercise 8	2
Ch.1, §1.1, Exercise 18	2
Ch.1, §1.1, Exercise 40	3
Chapter 1 Section 1.2	4
Ch.1, §1.2, Exercise 2	4
Ch.1, §1.2, Exercise 4	4
Ch.1, §1.2, Exercise 12	4
References	5

CHAPTER 1 SECTION 1.1

Ch.1, §1.1, Exercise 8. Write the following set by listing its elements between braces:

$$\{x \in \mathbb{R} : x^3 + 5x^2 = -6x\}.$$

1 \Rightarrow *Solution to Ch.1, §1.1, Exercise 8.* This solution was provided by Professor Casalaina.¹ For $x \in \mathbb{R}$, we have

$$\begin{aligned} x^3 + 5x^2 = -6x &\iff x^3 + 5x^2 + 6x = 0 \\ &\iff x(x^2 + 5x + 6) = 0 \\ &\iff x(x+2)(x+3) = 0 \\ &\iff x = 0, \text{ or } x = -2, \text{ or } x = -3. \end{aligned}$$

Therefore,

$$\{x \in \mathbb{R} : x^3 + 5x^2 = -6x\} = \{0, -2, -3\}.$$

□

Ch.1, §1.1, Exercise 18. Write the following set in set-builder notation:

$$\{0, 4, 16, 36, 64, 100, \dots\}.$$

Solution to Ch.1, §1.1, Exercise 18. This solution was provided by Professor Casalaina. We note that for $n = 0, \dots, 5$, we have the following values for $(2n)^2$:

n	$(2n)^2$
0	0
1	4
2	16
3	36
4	64
5	100

¹You are encouraged to work together on homework assignments. However, for each problem you must write out your own solution, and, as I have done here, you must indicate with whom you worked, and you must cite any resources you used in solving the problem. Plagiarism will not be tolerated – you will receive a 0 for the assignment.

Since this agrees with the list we were given, we may write

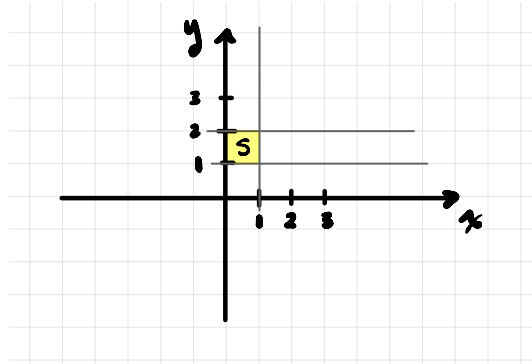
$$\{0, 4, 16, 36, 64, 100, \dots\} = \{(2n)^2 : n \in \mathbb{Z}_{\geq 0}\},$$

where $\mathbb{Z}_{\geq 0}$ is the set of integers that are greater than or equal to zero. □

Ch.1, §1.1, Exercise 40. Sketch the following set of points in the x, y -plane:

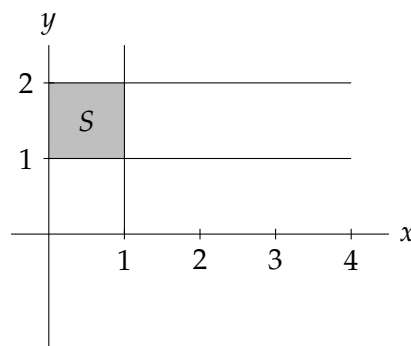
$$S = \{(x, y) : x \in [0, 1], y \in [1, 2]\}$$

Solution to Ch.1, §1.1, Exercise 40. This solution was provided by Professor Casalaina. For this problem I first sketched my own solution by hand, and then included my sketch:



However, it is also not too hard to implement the solution directly in \LaTeX , which typically will make it look better; I modified the `tikz`² code from the webpage:

<https://tex.stackexchange.com/questions/140312/tikz-shading-region-bounded-by-several-curves>



□

²tikz is a package I have included in this file.

CHAPTER 1 SECTION 1.2

Ch.1, §1.2, Exercise 2.*Solution to Ch.1, §1.2, Exercise 2.***Ch.1, §1.2, Exercise 4.***Solution to Ch.1, §1.2, Exercise 4.***Ch.1, §1.2, Exercise 12.***Solution to Ch.1, §1.2, Exercise 12.*

REFERENCES

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

UNIVERSITY OF COLORADO, DEPARTMENT OF MATHEMATICS, CAMPUS BOX 395, BOULDER, CO 80309

Email address: casa@math.colorado.edu