Exercise 2.10.4

Introduction to Discrete Mathematics
MATH 2001

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ABSTRACT. This is Exercise 2.10.4 from Hammack [Ham13, §2.10]:

Exercise 2.10.4. Negate the statement: *For every positive number ϵ, there is a positive number M for which $|f(x) - b| < \epsilon$ whenever $x > M$.*

Solution. First, I am going to rephrase the statement in more standard language. The letters ϵ, M, x, and b will denote real numbers, and $f(x)$ is a function (a real valued function of real numbers). Then the statement in the problem is:

For all $\epsilon > 0$ there exists $M > 0$ such that for all $x > M$ we have $|f(x) - b| < \epsilon$,

and the negation is:

There exists $\epsilon > 0$ such that for all $M > 0$ there exists $x > M$ such that $|f(x) - b| \geq \epsilon$. □

Remark 0.1. It may be helpful to phrase the statements above in more technical language. The statement in the problem can be written as

$$\forall \epsilon \in \{a \in \mathbb{R} : a > 0\}, \ \exists M \in \{a \in \mathbb{R} : a > 0\}, \ \forall x \in \{a \in \mathbb{R} : a > M\}, \ |f(x) - b| < \epsilon.$$

Then the negation is

$$\exists \epsilon \in \{a \in \mathbb{R} : a > 0\}, \ \forall M \in \{a \in \mathbb{R} : a > 0\}, \ \exists x \in \{a \in \mathbb{R} : a > M\}, \ |f(x) - b| \geq \epsilon.$$

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References

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

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