Exercise 10.1

Introduction to Discrete Mathematics
MATH 2001

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ABSTRACT. This is Exercise 10.1 from Hammack [Ham13, Ch. 10]:

Exercise 10.1. Prove that for each natural number n we have

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

Solution. For each natural number n, we consider the statement

$$p(n) : \sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

We want to prove by induction that for all natural numbers n, the statement $p(n)$ is true.

The first step is to show that the statement $p(1)$ is true. The statement $p(1)$ is the statement

$$p(1) : \sum_{i=1}^{1} i = \frac{1(1+1)}{2}$$

which is true, since $\sum_{i=1}^{1} i = 1 = \frac{1(1+1)}{2}$.

Now let N be a natural number, and assume that we have proven the statement $p(n)$ for all natural numbers n less than or equal to N. We must show that under these assumptions, the statement $p(N+1)$ is also true.

The statement $p(N+1)$ is the statement

$$p(N+1) : \sum_{i=1}^{N+1} i = \frac{(N+1)(N+2)}{2}.$$

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Let us now confirm that $p(N+1)$ is true. We have

$$\begin{aligned}
\sum_{i=1}^{N+1} i &= \sum_{i=1}^{N} i + (N+1) \\
&= \frac{N(N+1)}{2} + (N+1) && \text{(Using that } p(N) \text{ is assumed to be true.)} \\
&= \frac{N(N+1)}{2} + \frac{2(N+1)}{2} \\
&= \frac{N^2+3N+2}{2} = \frac{(N+1)(N+2)}{2}.
\end{aligned}$$

This completes the proof. □

References

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

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