# Midterm 2

## Abstract Algebra 1

## MATH 3140

## Fall 2022

Friday October 28, 2022

**UPLOAD THIS COVER SHEET!**

NAME: ______________________________

PRACTICE EXAM

| Question: | **1** | **2** | **3** | **4** | **5** | Total |
|---|---|---|---|---|---|---|
| Points: | 20 | 20 | 20 | 20 | 20 | 100 |
| Score: | | | | | | |

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with the questions in the correct order, etc.
- You have 45 minutes to complete the exam.

**1.** (a) (5 points) • *Is the permutation* $\sigma = (1,6,4)(2,5) \in S_6$ *even or odd?*

(b) (5 points) *Is the permutation* $\sigma^2$ *even or odd?*

(c) (5 points) *Compute* $|\sigma|$; i.e., the order of the element $\sigma$ in the group $S_6$.

(d) (5 points) *With* $\sigma$ *as above and* $\tau = (5,3,2)$, *compute* $\sigma\tau$ *(as a product of disjoint cycles).*

| 1 |
|---|
| |
| 20 points |

**2.** • Consider the dihedral group $D_n$, with $n \geq 3$. Recall the notation we have been using: $D_n$ has identity element $I$, and is generated by elements $R$ and $D$, satisfying the relations $R^n = D^2 = I$ and $RD = DR^{-1}$. Consider the cyclic subgroup $\langle R^2 \rangle$.

(a) (10 points) *Show that* $\langle R^2 \rangle$ *is a normal subgroup of* $D_n$.

(b) (10 points) *Find the order of the group* $D_n / \langle R^2 \rangle$. [*Hint:* this may depend on the parity of $n$.]

| 2 |
| --- |
| 20 points |

**3.** • Recall that for a commutative ring $R$ with unity $1 \neq 0$, we define $R[x]$ to be the ring of polynomials in $x$ with coefficients in $R$. Consider the map

$$\phi : \mathbb{Z}[x] \longrightarrow \mathbb{Z}_4[x]$$

$$\sum_{k=0}^{n} a_k x^k \mapsto \sum_{k=0}^{n} [a_k] x^k,$$

where $[a_k] = a_k \pmod 4$.

(a) (10 points) *Show that $\phi$ is a homomorphism of rings.*

(b) (10 points) *Describe the kernel of $\phi$.* (Do not just write down the definition; you need to describe an explicit subset of $\mathbb{Z}[x]$.)

| 3 |
| --- |
| 20 points |

**4.** (20 points) • In a commutative ring with unity, *show that* $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$.

| 4 |
| --- |
| 20 points |

**5.** • **TRUE** or **FALSE**. For this problem, and this problem only, **you do not need to justify your answer**.

(a) (4 points) **TRUE** or **FALSE** (circle one). The order of an element of a finite group divides the order of the group.

(b) (4 points) **TRUE** or **FALSE** (circle one). The symmetric group $S_n$ is not cyclic for any $n \geq 1$.

(c) (4 points) **TRUE** or **FALSE** (circle one). Every abelian group of order divisible by 5 contains a cyclic subgroup of order 5.

(d) (4 points) **TRUE** or **FALSE** (circle one). Every quotient group ("factor group") of a cyclic group is cyclic.

(e) (4 points) **TRUE** or **FALSE** (circle one). If $F$ is a field, and $R$ is a subring of $F$ with unity $1_R$ in $R$ equal to unity $1_F$ in $F$, then $R$ is a field.

| 5 |
|---|
| |
| 20 points |