Exercise 7.2

Abstract Algebra 1
MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 7.2 from Fraleigh [Fra03, §7]:

Exercise 7.2. List the elements of the subgroup of $\mathbb{Z}_{12}$ generated by the subset $\{4, 6\}$.

Solution. The subgroup of $\mathbb{Z}_{12}$ generated by $\{4, 6\}$ is the subgroup $\{0, 2, 4, 6, 8, 10\}$. Indeed, since $\gcd(4, 6) = 2$, the subgroup of $\mathbb{Z}_{12}$ generated by the subset $\{4, 6\}$ is the same as the subgroup generated by 2 (or more directly, since $2 = 6 - 4$, the subgroup generated by 2 is contained in the subgroup generated by $\{4, 6\}$, and since 4 and 6 are in the subgroup generated by 2, the subgroup generated by 2 is equal to the subgroup generated by $\{4, 6\}$). The subgroup of $\mathbb{Z}_{12}$ generated by 2 is the subgroup $\{0, 2, 4, 6, 8, 10\}$. □

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REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

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