Exercise 0.30

Abstract Algebra 1
MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 0.30 from Fraleigh [Fra03, §0]:

Exercise 0.30. Determine whether the relation

$$x\mathscr{R}y \text{ in } \mathbb{R} \text{ if } x \geq y$$

is an equivalence relation on $\mathbb{R}$. If so, describe the partition arising from the equivalence relation.

Note that as a set, the relation is defined as:

$$\mathscr{R} = \{(x,y) \in \mathbb{R}^2 : x \geq y\} \subseteq \mathbb{R}^2.$$

Solution. The relation $\mathscr{R}$ is not an equivalence relation since it does not satisfy the symmetric property. For instance,

$$2\mathscr{R}1,$$

but it is *not* true that $1\mathscr{R}2$. In other words, $(2,1) \in \mathscr{R}$ since $2 \geq 1$, but $(1,2) \notin \mathscr{R}$ since $1 \not\geq 2$. □

Remark 0.1. Note that $\mathscr{R}$ *does* satisfy the reflexive and transitive properties. Indeed, if $x \in \mathbb{R}$, then $(x,x) \in \mathscr{R}$ (i.e., $x \geq x$), so that $\mathscr{R}$ satisfies the reflexive property. And if $x, y, z \in \mathbb{R}$, with $(x,y) \in \mathscr{R}$ (i.e., $x \geq y$) and $(y,z) \in \mathscr{R}$ (i.e., $y \geq z$), then $(x,z) \in \mathscr{R}$ (i.e., $x \geq z$), so that $\mathscr{R}$ satisfies the transitive property.

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References

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

University of Colorado, Department of Mathematics, Campus Box 395, Boulder, CO 80309

Email address: `casa@math.colorado.edu`