Midterm 2

Linear Algebra: Matrix Methods

MATH 2130

Fall 2022

Friday October 28, 2022

UPLOAD THIS COVER SHEET!

NAME: ______________________________

PRACTICE EXAM

Question:	**1**	**2**	**3**	**4**	**5**	Total
Points:	20	20	20	20	20	100
Score:						

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with the questions in the correct order, etc.
- You have 45 minutes to complete the exam.

1. • *Compute the determinant of each of the following matrices:*

(a) (10 points) $A = \begin{pmatrix} 4 & -1 & 1 \\ -1 & -2 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

(b) (10 points) $B = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \pi \\ 1 & 0 & e & -4 & 8 & 3^{-5} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 5 & 1 & 0 & 2 & 10^4 \\ 0 & 0 & 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & -1 & 2 & 0 \end{pmatrix}$

1
20 points

2. (20 points) • Let V_1 and V_2 be real vector spaces. On the product

$$V_1 \times V_2 = \{(\mathbf{v}_1, \mathbf{v}_2) : \mathbf{v}_1 \in V_1,\ \mathbf{v}_2 \in V_2\},$$

define addition and scaling rules by

$$(\mathbf{v}_1, \mathbf{v}_2) + (\mathbf{w}_1, \mathbf{w}_2) = (\mathbf{v}_1 + \mathbf{w}_1, \mathbf{v}_2 + \mathbf{w}_2)$$

$$\lambda \cdot (\mathbf{v}_1, \mathbf{v}_2) = (\lambda \cdot \mathbf{v}_1, \lambda \cdot \mathbf{v}_2).$$

Show that these addition and scaling rules make $V_1 \times V_2$ into a real vector space.

2
20 points

3. (20 points) • Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ be bases for a real vector space V, and suppose that

$$\begin{aligned} \mathbf{v}_1 &= 4\mathbf{w}_1 - \mathbf{w}_2 + \mathbf{w}_3 \\ \mathbf{v}_2 &= 3\mathbf{w}_1 + 2\mathbf{w}_2 - \mathbf{w}_3 \\ \mathbf{v}_3 &= 7\mathbf{w}_1 + 23\mathbf{w}_2 - 2\mathbf{w}_3 \end{aligned}$$

Find the change-of-coordinates matrix to go from the coordinates with respect to the basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ *to the coordinates with respect to the basis* $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$.

3
20 points

4. • Consider the two dimensional discrete dynamical system

$$\mathbf{x}_{k+1} = A\mathbf{x}_k$$

where

$$A = \begin{pmatrix} 1.7 & 0.3 \\ 1.2 & 0.8 \end{pmatrix}$$

(a) (10 points) *Is the origin an attractor, repeller, or saddle point?*

(b) (10 points) *Find the directions of greatest attraction or repulsion.*

4
20 points

5. • Consider the following real matrix

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & -1 \\ 2 & 1 & 3 \end{pmatrix}$$

(a) (5 points) *Find the characteristic polynomial $p_A(t)$ of A.*

(b) (5 points) *Find the eigenvalues of A.*

(c) (5 points) *Find a basis for each eigenspace of A in $\mathbb{R}^3$.*

(d) (5 points) *Is A diagonalizable? If so, find a matrix $S \in \mathrm{M}_{3\times 3}(\mathbb{R})$ so that $S^{-1}AS$ is diagonal. If not, explain*.

5
20 points