Midterm 1

Linear Algebra: Matrix Methods

MATH 2130

Fall 2022

Friday September 23, 2022

NAME: ________________________________

PRACTICE EXAM

Question:	**1**	**2**	**3**	**4**	**5**	Total
Points:	20	10	30	20	20	100
Score:						

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with the questions in the correct order, etc.
- You have 45 minutes to complete the exam.

1. (20 points) • *Find all solutions to the following system of linear equations:*

$$\begin{aligned} 3x_1 &+ 9x_2 &+ 27x_3 &= -3 \\ -3x_1 &- 11x_2 &- 35x_3 &= 5 \\ 2x_1 &+ 8x_2 &+ 26x_3 &= -4 \end{aligned}$$

1
20 points

2. (10 points) • Consider the linear map ("transformation") $L : \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$L(x_1, x_2, x_3) = (2x_1 - x_3, 3x_2 + x_3).$$

Write down the matrix form of ("standard matrix for") the linear map L.

2
10 points

3. • Consider the following matrix A and its corresponding Reduced Row Echelon Form matrix RREF(A):

$$A = \begin{bmatrix} 1 & -3 & 0 & -1 & 4 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \\ 3 & -9 & 0 & -3 & 2 & 4 \\ 1 & -3 & 1 & -2 & 4 & -1 \end{bmatrix} \quad \text{RREF}(A) = \begin{bmatrix} 1 & -3 & 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (3 points) *Are the rows of A linearly independent?*

(b) (3 points) *Are the columns of A linearly independent?*

(c) (3 points) *What is the row rank of A?*

(d) (3 points) *What is the column rank of A?*

(e) (6 points) *Find a basis for the row space of A.*

(f) (6 points) *Find a basis for the column space of A.*

(g) (6 points) *Find a basis for the space of solutions of the matrix equation* $A\mathbf{x} = \mathbf{0}$.

3
30 points

4. • Consider the matrix

$$B = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

(a) (10 points) *Find the inverse of B.*

(b) (10 points) *Does there exist* $\mathbf{x} \in \mathbb{R}^3$ *such that* $B\mathbf{x} = \begin{bmatrix} 5 \\ \sqrt{2} \\ \pi \end{bmatrix}$*?*

4
20 points

5. (20 points) • The equation

$$\mathbf{x} = C\mathbf{x} + \mathbf{d}$$

(the *Leontief Production Equation*) arises in the Leontief Input-Output Model. Here $\mathbf{x}, \mathbf{d} \in M_{n\times 1}(\mathbb{R})$ are column vectors and $C \in M_{n\times n}(\mathbb{R})$ is a square matrix. Consider also the equation $\mathbf{p} = C^T\mathbf{p} + \mathbf{v}$ (called the *Price Equation*), where $\mathbf{p}, \mathbf{v} \in M_{n\times 1}(\mathbb{R})$ are column vectors.

Show that

$$\mathbf{p}^T\mathbf{d} = \mathbf{v}^T\mathbf{x}.$$

(This quantity is known as GDP.) [*Hint:* Compute $\mathbf{p}^T\mathbf{x}$ in two ways.]

5
20 points