# HOMEWORK 1

# LINEAR ALGEBRA

# MATH 2130

SEBASTIAN CASALAINA

ABSTRACT. This is Homework 1. The problems are from Lay [LLM16, §1.1–2]:

- HW1a Lay Section 1.1: 1, 3, 16.
- HW1a Lay Section 1.2: 1, 7, 29.

## CONTENTS

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<div align="center">SECTION 1.1</div>

**Exercise 1.1.1.** Solve the system by using elementary row operations on the equations or on the augmented matrix. Follow the systematic elimination procedure described in this section.

$$\begin{aligned} x_1 + 5x_2 &= 7 \\ -2x_1 - 7x_2 &= -5 \end{aligned}$$

*Solution.* The associated augmented matrix for the system of equations is:

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right]$$

Adding 2 times the first row to the second row gives:

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array}\right]$$

Dividing the second row by 3 gives

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 1 & 3 \end{array}\right]$$

Adding $-5$ times the second row to the first row gives

$$\left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & 3 \end{array}\right]$$

Therefore, the solution is

$$\boxed{x_1 = -8,\ x_2 = 3.}$$

$\square$

*Remark* 0.1. While it is not strictly necessary for the problem, note that we can check that the solution $x_1 = -8$, $x_2 = 3$ is correct by plugging back into the original equations:

$$\begin{aligned} (-8) + 5(3) &= 7 \\ -2(-8) - 7(3) &= -5 \end{aligned}$$

**Exercise 1.1.3.**

*Solution.* $\square$

**Exercise 1.1.16.**

*Solution.*                                                                    □

SECTION 1.2

**Exercise 1.2.1.** Determine which matrices are in reduced (row) echelon form, and which others are only in (row) echelon form.

a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$  b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$  c. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$  d. $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

*Solution.* a. RREF, b. RREF, c. Neither (it has a row of zeros above a non-zero row), d. REF.  □

**Exercise 1.2.7.**

*Solution.*  □

**Exercise 1.2.29.**

*Solution.*  □

## REFERENCES

[LLM16]  David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Fifth edition, Pearson, 2016.

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