

Exercise 4.2.26

**Linear Algebra
MATH 2130**

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ABSTRACT. This is Exercise 4.2.26 from Lay [LLM16, §4.2]:

Exercise 4.2.26. True or False. Justify each answer.

- (a) A kernel (“**null space**”) is a vector space.

Solution. TRUE. Given a linear map $L : V \rightarrow V'$ of vector spaces, we have seen that the kernel of L is a sub-space of V , and therefore is a vector space (see also [LLM16, Theorem 2, p.201]). □

- (b) The column space of an $m \times n$ real matrix is in \mathbb{R}^m .

Solution. TRUE. The columns of the matrix have length m . □

- (c) $\text{Col } A$ is the set of all solutions of $A\mathbf{x} = \mathbf{b}$.

Solution. FALSE. For instance if A is an $m \times n$ matrix, with $m \neq n$, then the column space of A is a (nonempty) subspace of \mathbb{R}^m , while the set of all solutions of $A\mathbf{x} = \mathbf{b}$ form a subset of \mathbb{R}^n , and so the column space of A cannot be equal to the set of all solutions of $A\mathbf{x} = \mathbf{b}$.

For reference, the column space of A is the set of all \mathbf{b} such there exists a solution to the matrix equation $A\mathbf{x} = \mathbf{b}$ (see also the bottom of [LLM16, p.203]). □

- (d) $\ker A$ (“**Nul } A**”) is the kernel of the mapping $\mathbf{x} \mapsto A\mathbf{x}$.

Solution. TRUE. This is the definition (see also [LLM16, p.201]). □

(e) The image (“range”) of a linear map (“transformation”) is a vector space.

Solution. TRUE. We have seen that the image of a linear map is a vector space (see also [LLM16, p.206]). □

(f) The set of all solutions of a homogeneous linear differential equation is the kernel of a linear map (“transformation”).

Solution. TRUE. See the bottom of [LLM16, p.206]. □

REFERENCES

[LLM16] David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Fifth edition, Pearson, 2016.

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