Midterm II

Abstract Algebra 1

MATH 3140

Fall 2021

Friday October 29, 2021

NAME: ______________________________

PRACTICE EXAM

Question:	**1**	**2**	**3**	**4**	**5**	Total
Points:	20	20	20	20	20	100
Score:						

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- Either write your solutions **directly on this exam** or write the solution to **each problem on a separate piece of paper**.
- You must upload your exam as a single **.pdf** to **Canvas**, with the questions in the correct order, etc.
- You have 50 minutes to complete the exam. **Do not forget to leave yourself time (at least 5 minutes) at the end to upload your exam.**

1. (a) (5 points) • *Is the permutation* $\sigma = (1,6,4)(2,5) \in S_6$ *even or odd?*

(b) (5 points) *Is the permutation* σ^2 *even or odd?*

(c) (5 points) *Compute* $|\sigma|$; i.e., the order of the element σ in the group S_6.

(d) (5 points) *With* σ *as above and* $\tau = (5,3,2)$, *compute* $\sigma\tau$ *(as a product of disjoint cycles).*

1
20 points

2. • Let A be a set, and let $G \leq S_A$ be a subgroup of the group of permutations S_A of A. For an element $a \in A$, define $G_a := \{\sigma \in G : \sigma(a) = a\}$.

(a) (10 points) *For $a \in A$, show that G_a is a subgroup of G.*

(b) (10 points) Let $a, b \in A$, and suppose there exists $\sigma \in G$ such that $b = \sigma(a)$. *Show that G_a and G_b have the same cardinality.*

2
20 points

3. • Consider the dihedral group D_n, with $n \geq 3$. Recall the notation we have been using: D_n has identity element I, and is generated by elements R and D, satisfying the relations $R^n = D^2 = I$ and $RD = DR^{-1}$. Consider the cyclic subgroup $\langle R^2 \rangle$.

(a) (10 points) *Show that $\langle R^2 \rangle$ is a normal subgroup of D_n.*

(b) (10 points) *Find the order of the group $D_n / \langle R^2 \rangle$.* [*Hint:* this may depend on the parity of n.]

3
20 points

4. • Recall that for a commutative ring R with unity $1 \neq 0$, we define $R[x]$ to be the ring of polynomials in x with coefficients in R. Consider the map

$$\phi : \mathbb{Z}[x] \longrightarrow \mathbb{Z}_4[x]$$

$$\sum_{k=0}^{n} a_k x^k \mapsto \sum_{k=0}^{n} [a_k] x^k,$$

where $[a_k] = a_k \pmod 4$.

(a) (10 points) *Show that ϕ is a homomorphism of rings.*

(b) (10 points) *Describe the kernel of ϕ.* (Do not just write down the definition; you need to describe an explicit subset of $\mathbb{Z}[x]$.)

4
20 points

5. (20 points) • In a commutative ring with unity, *show that* $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$.

5
20 points