Take-Home Final

Abstract Algebra 1

MATH 3140

Fall 2021

Sunday December 12, 2021

NAME: _____

PRACTICE EXAM

Question:	**1**	**2**	**3**	**4**	Total
Points:	25	25	25	25	100
Score:					

- For the exam you may use **only the following resources**: our textbook, your lecture notes, my lecture notes, your homework, the pdfs linked from the course webpage:

  `http://math.colorado.edu/~casa/teaching/21fall/3140/hw.html`

  and the quizzes and midterms we have taken on Canvas.
- You **may not use any other resources** whatsoever.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam to **Canvas** as a **single .pdf** file with the questions in the correct order.
- The exam is due at 12:00 PM (noon) December 12, 2021.

1. (25 points) • Let G be a group with center $Z(G)$. *Show that if* $G/Z(G)$ *is cyclic, then* $Z(G) = G$.

[*Hint:* Show first there exists $g \in G$ such that for any $g_1 \in G$, there is a $z_1 \in Z(G)$ and $n_1 \in \mathbb{Z}$ such that $g_1 = g^{n_1} z_1$. Then show for any $g_1, g_2 \in G$ that $g_1 g_2 = g_2 g_1$.]

1
25 points

2. (25 points) • **True or False**: *There exist a ring R with unity $1 \neq 0$, a ring R' with unity $1' \neq 0'$, and homomorphism of rings $\phi : R \to R'$ such that $\phi(1) \neq 0'$ and $\phi(1) \neq 1'$.*

2
25 points

3. (25 points) • Let D be an integral domain, and suppose that for every descending chain of ideals in D

$$\cdots \subseteq I_4 \subseteq I_3 \subseteq I_2 \subseteq I_1 \subseteq D$$

there is a positive integer n such that $I_m = I_n$ for all $m \geq n$. *Show that D is a field.*

3
25 points

4. (25 points) • *Show that if F, E, and K are fields with $F \leq E \leq K$, then K is algebraic over F if and only if E is algebraic over F, and K is algebraic over E.* (You must *not* assume the extensions are finite.)

4
25 points