

In-Class Final

Abstract Algebra 1

MATH 3140

Fall 2021

Sunday December 12, 2021

NAME: _____

PRACTICE EXAM

Question:	1	2	3	4	Total
Points:	25	25	25	25	100
Score:					

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam to **Canvas** as a **single .pdf** file with the questions in the correct order.
- You have 60 minutes to complete the exam.

1. (25 points) • Show that for a prime p , the polynomial $x^p + a \in \mathbb{Z}_p[x]$ is not irreducible for any $a \in \mathbb{Z}_p$.

1
25 points

2. (25 points) • Let R be a commutative ring and let I be an ideal of R . The *radical of I* is the set

$$\sqrt{I} := \{a \in R : a^n \in I \text{ for some } n \in \mathbb{Z}^+\}.$$

Show that \sqrt{I} is an ideal of R .

2
25 points

3. (25 points) • Prove that the algebraic closure of \mathbb{Q} in \mathbb{C} is not a finite extension of \mathbb{Q} .

3
25 points

4. (25 points) • Find the degree and a basis for the field extension $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over \mathbb{Q} .

4
25 points