Exercise 2.2

Abstract Algebra 1
MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 2.2 from Fraleigh [Fra03, §2]:

Exercise 2.2. The binary operation $*$ is defined on $S = \{a, b, c, d\}$ by means of the table [Fra03, 2.26 Table, p.26]:

$*$	a	b	c	d	e
a	a	b	c	b	d
b	b	c	a	e	c
c	c	a	b	b	a
d	b	e	b	e	d
e	e	b	a	d	c

Compute $(a * b) * c$ and $a * (b * c)$. Can you say on the basis of this computation whether $*$ is associative?

Solution. We have

$$(a * b) * c = b * c$$
$$= a$$
$$a * (b * c) = a * a$$
$$= a$$

While $(a * b) * c = a * (b * c)$, we cannot determine based only on this computation whether $*$ is associative. For that, we must check whether *for all* $x, y, z \in S$ we have $(x * y) * z = x * (y * z)$; we have only checked this for $x = a$, $y = b$, and $z = c$. □

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Remark 0.1. Note that, in fact, $*$ is *not* associative. Indeed, we have for instance

$$\begin{aligned}(d*d)*e &= e*d \\ &= d \\ d*(d*e) &= d*d \\ &= e\end{aligned}$$

Since $(d*d)*e \neq d*(d*e)$, we have that $*$ is not associative.

REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

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