Exercise 11.47

Abstract Algebra 1
MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 11.47 from Fraleigh [Fra03, §11]:

Exercise 11.47. Let G be an abelian group. Let H be the subset of G consisting of the identity e together with all elements of order 2. Show that H is a subgroup of G.

Solution. Let G be an abelian group. Let H be the subset of G consisting of the identity e together with all elements of order 2. To show that H is a subgroup, it suffices to show that H is nonempty, and for all $a, b \in H$, one has $ab^{-1} \in H$. Since $e \in H$, we have that H is nonempty. So let $a, b \in H$. Then

$$(ab^{-1})(ab^{-1}) = ab^{-1}ab^{-1} = aab^{-1}b^{-1} = aa(bb)^{-1} = ee = e.$$

Thus $ab^{-1} \in H$. $\square$

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References

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

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