Exercise 10.40

Abstract Algebra 1
MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 10.40 from Fraleigh [Fra03, §10]:

Exercise 10.40. Let G be a finite group of order n with identity e. Show that for any $a \in G$, we have $a^n = e$.

Solution. Let G be a finite group of order n with identity e, and let $a \in G$. We know that the order of a is finite; indeed the cyclic group $\langle a \rangle$ is a subgroup of the finite group G, and therefore must be finite (alternatively, see [Fra03, Exercise 4.34]). Let $r = |a|$ be the order of a, which we have seen is the smallest positive integer r such that $a^r = e$ (see the pdf on the webpage, explaining the assertion on [Fra03, §6, p.59]). From [Fra03, Theorem 10.12, p.101] we know that r divides n; in other words, $n = rs$ for some natural number s. From this we have $a^n = a^{rs} = (a^r)^s = e^s = e$. $\square$

Date: October 5, 2021.

REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

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