MIDTERM II
LINEAR ALGEBRA

MATH 2135

Friday March 23, 2018.

Name | ______________________________

PRACTICE EXAM

Please answer the all of the questions, and show your work.
You must explain your answers to get credit.
You will be graded on the clarity of your exposition!

1	2	3	4	5	6	
20	20	20	20	20	20	total

Date: March 19, 2018.

1
20 points

1. Let V be an n-dimensional vector space over $K \in \{\mathbb{Q}, \mathbb{R}, \mathbb{C}\}$, and let $v_1, \ldots, v_n$ be a basis for V. *Give the definition of a determinant function for the vector space V with respect to the basis $v_1, \ldots, v_n$.*

2
20 points

2. Let V be the real vector space spanned by $1, \cos t, \sin t$ in the real vector space $\mathrm{Diff}(\mathbb{R}, \mathbb{R})$ of differentiable real valued functions.

2.(a). Let $T : V \to V$ be the linear map defined by differentiation, i.e., $T(f) = f'$. *Give the matrix form of T with respect to the basis* $1, \cos t, \sin t$.

2.(b). *Find two bases v_1, v_2, v_3 and w_1, w_2, w_3 for V so that with respect to these bases, the matrix form of T is diagonal.*

More precisely, find two bases v_1, v_2, v_3 and w_1, w_2, w_3 for V so that if the first basis defines an isomorphism $\phi : \mathbb{R}^3 \to V$ and the second defines an isomorphism $\psi : \mathbb{R}^3 \to V$, then the matrix associated to the composition

$$\mathbb{R}^3 \xrightarrow{\phi} V \xrightarrow{T} V \xrightarrow{\psi^{-1}} \mathbb{R}^3$$

is diagonal.

3. *Find the reduced row echelon form of the following matrix:*

3
20 points

$$A = \begin{pmatrix} 1 & -3 & 0 & -1 & 4 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \\ 3 & -9 & 0 & -3 & 2 & 4 \\ 1 & -3 & 1 & -2 & 4 & -1 \end{pmatrix}$$

4. Let A be the matrix in the previous problem.

4
20 points

4.(a). Let $A^T : \mathbb{R}^4 \to \mathbb{R}^6$ be the linear map associated to the transpose of A. *Find a basis for the image of A^T.*

4.(b). Let $A : \mathbb{R}^6 \to \mathbb{R}^4$ be the linear map associated to A. *Find a basis for the kernel of A.*

4.(c). *Find all real solutions to the system of linear equations:*

$$\begin{array}{rcrcrcrcrcr} x_1 & - & 3x_2 & & & - & x_4 & + & 4x_5 & = & -2 \\ & & & & x_3 & - & x_4 & & & = & 1 \\ 3x_1 & - & 9x_2 & & & - & 3x_4 & + & 2x_5 & = & 4 \\ x_1 & - & 3x_2 & + & x_3 & - & 2x_4 & + & 4x_5 & = & -1 \end{array}$$

5. *Find the determinant of the following matrix:*

5
20 points

$$B = \begin{pmatrix} 1 & -3 & 2 & 0 & -1 \\ 0 & 3 & 1 & -1 & 1 \\ 2 & -6 & 2 & -3 & 2 \\ 0 & 3 & 3 & -2 & 2 \\ 0 & -3 & 1 & 1 & 1 \end{pmatrix}$$

6
20 points

6. *For* $K \in \{\mathbb{Q}, \mathbb{R}, \mathbb{C}\}$, *show:*

A matrix $A \in \mathrm{M}_{n\times n}(K)$ has rank $n-1 \iff \det A = 0$ and there is some $(n-1)\times(n-1)$ minor A_{ij} of A with $\det A_{ij} \neq 0$.