Math 2300-007: Quiz 6

Name: Solutions 2/27/18 Score: _____

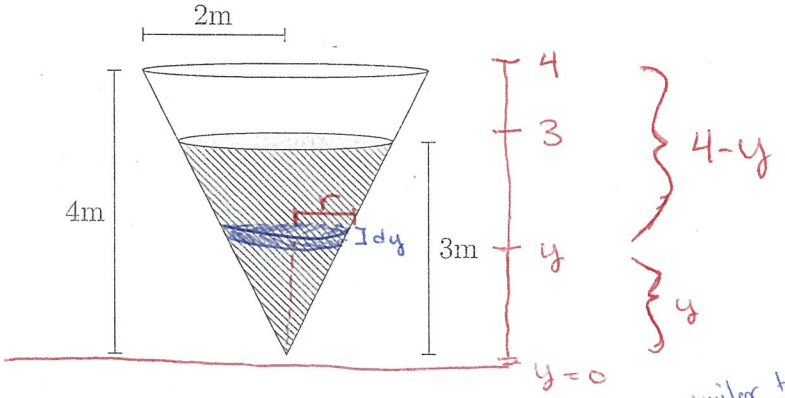
1. (1 point) Write down a formula for work in terms of force and distance.

$W = F \cdot d$

2. (1 point) In a sentence or two, describe your plan for solving the problem below.

e.g. Find work to lift one slice. Use integral to add up all of the small slices of work.

3. (5 points) Water is pumped from the top of a conical tank of height 4 meters and base radius 2 meters depicted below. How much work is required to empty the tank of water if the initial height of the water is 3 meters? You may assume that the mass of water on Earth is 1000 kilograms per cubic meter and that $g = 9.8 \frac{\text{m}}{\text{sec}^2}$ is the acceleration due to gravity on Earth. **(Set up, but do not evaluate the integral.)**



$W_{slice} = F_g \cdot d$

$= m \cdot g \cdot (4-y)$

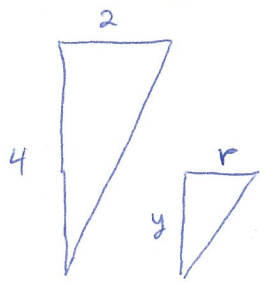
$= V_{slice} \cdot 10^3 \cdot g \cdot (4-y)$

$= \pi r^2 dy \cdot 10^3 g (4-y)$

$= \pi \left(\frac{y}{2}\right)^2 dy \cdot 1000 g (4-y)$

To find r, use similar triangles or eqn of the line.

Similar triangles:



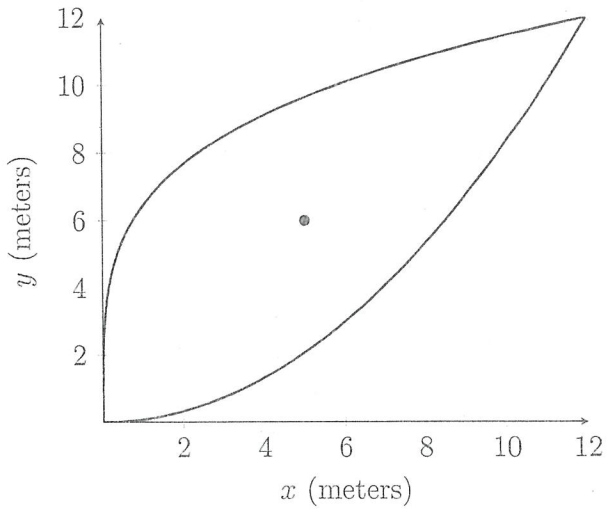
$\frac{4}{2} = \frac{y}{r}$, so $y = 2r$, $r = \frac{y}{2}$.

(Could also use that the equation of the "line" is $y = 2x$, where $r = x$.)

Total Work $= \int_{y=0}^{y=3} W_{slice}$ slices "live" between 0 & 3

$= \int_0^3 1000\pi g \cdot (4-y)\left(\frac{y}{2}\right)^2 dy$

4. The lamina depicted below has centroid $(5, 6)$ and mass density $\rho = 1$ kilogram per square meter. Suppose M_x is the moment of the system about the x-axis and M_y is the moment of the system about the y-axis.



Use the figure provided to answer the following questions. For each question, choose the best answer.

(a) (1 point) If the density is changed from $\rho = 1$ kg/m^2 to $\rho = 2$ kg/m^2, then:

(i) M_x increases, M_y increases, and the centroid stays at $(5, 6)$;
(ii) M_x increases, M_y decreases, and the centroid stays at $(5, 6)$;
(iii) M_x increases, M_y increases, and the centroid moves to the right of $(5, 6)$;
(iv) M_x decreases, M_y increases, and the centroid moves to the left of $(5, 6)$;
(v) M_x decreases, M_y decreases, and the centroid stays at $(5, 6)$.
(vi) M_x and M_y do not change, and the centroid stays at $(5, 6)$.

(b) (1 point) If a point mass of 3 kg is added to the system at the point $(7, 6)$, then:

(i) M_x increases, M_y does not change, and the centroid stays at $(5, 6)$;
(ii) M_x increases, M_y decreases, and the centroid stays at $(5, 6)$;
(iii) M_x increases, M_y increases and the centroid moves to the right of $(5, 6)$;
(iv) M_x decreases, M_y does not change and the centroid moves to the left of $(5, 6)$;
(v) M_x and M_y do not change, and the centroid stays at $(5, 6)$.