Math 2300-007: Quiz 13/14

Name: Solutions 4/29/18 Score: _____

1. A ray (a half-line) is parameterized by $\begin{cases} x = 2 + 3t^2 \\ y = -1 + 4t^2, \end{cases}$ where $-\infty < t < \infty$.

(a) (2 points) Does $(2, 1)$ lie on the ray? Explain.

$2 = x = 2 + 3t^2 \Rightarrow 0 = 3t^2 \Rightarrow t = 0$, so the only time when $x = 2$ is $t = 0$.

On the other hand, $y(0) = -1 + 4(0)^2 = -1 \neq 1$, so there is no time when the line hits $(2, 1)$.

(b) (2 points) What is the speed of motion along the line as a function of t?

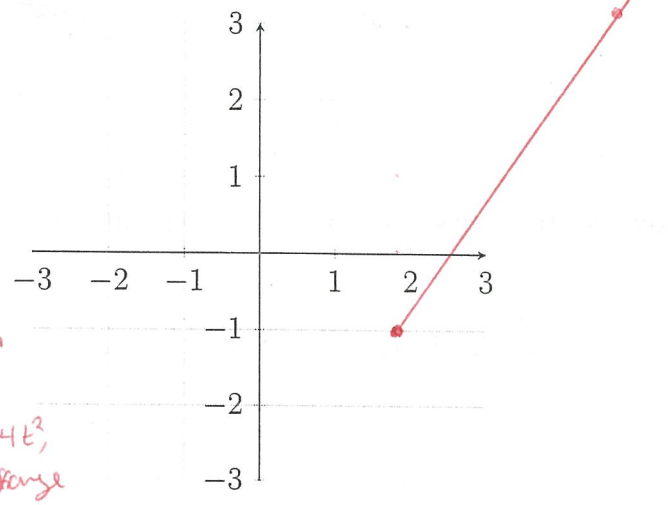
$$\text{Speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(6t)^2 + (8t)^2} = \sqrt{36t^2 + 64t^2}$$

$$= \sqrt{100t^2} = 10|t|$$

(c) (2 points) What is the slope of the line?

$$\text{Slope} = \frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{8t}{6t} = \frac{4}{3}$$

(d) (1 point) On the axes provided, draw the graph of the ray.



Note: Since $x = 2 + 3t^2$, the x-values range from 2 to ∞. (Similarly, $y = -1 + 4t^2$, so the y-values range from -1 to ∞.)

2. (3 points) Eliminate the parameter to find a cartesian equation of the curve described by the parametric equations

$$\begin{cases} x = 3\sin(t) \\ y = \cos(t) - 1, \end{cases} \qquad \text{where } 0 \le t < 2\pi.$$

You do not need to solve for x or y in your answer.

Notice that $\sin^2(t) + \cos^2(t) = 1$ for any t. This will help us "eliminate" the parameter:

$$\begin{cases} \frac{x}{3} = \sin t \\ y+1 = \cos t \end{cases} \Rightarrow \begin{cases} \left(\frac{x}{3}\right)^2 = \sin^2 t \\ (y+1)^2 = \cos^2 t \end{cases}$$

Hence,

$$\left(\frac{x}{3}\right)^2 = \sin^2 t$$
$$+ \quad (y+1)^2 = \cos^2 t$$
$$\left(\frac{x}{3}\right)^2 + (y+1)^2 = 1 \quad \leftarrow \text{Cartesian equation.}$$

3. (**Quiz 14**: 10 points) This weekend, something fun I'm doing is....

Mountain Biking!