Math 2300-007: Quiz 12

Name: Solutions 4/30/18 Score: _____

1. (6 points) A population $P(t)$ grows according to a logistic model and satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{4}{10}P\left(1 - \frac{P}{400}\right), \quad P(0) = 10,$$

where t is measured in years.

(a) What is the carrying capacity in this situation?

400 (As $P \to 400$, the rate of growth $\frac{dP}{dt}$ slows to 0)

(b) What is $P'(0)$? When $t=0$, $P=10$ by initial condition:

$$\left.\frac{dP}{dt}\right|_{t=0} = \left.\frac{dP}{dt}\right|_{P=10} = \frac{4}{10}\cdot 10\left(1 - \frac{10}{400}\right) = 4\left(1 - \frac{1}{40}\right)$$

$$= 4\cdot\frac{39}{40} = \frac{39}{10} = 3.9$$

(c) Interpret the meaning of $P'(0)$. Mention the units in your answer.

At time $t=0$, the population is growing at a rate of 3.9 individuals per year.

2. (4 points) Suppose $P(t)$ represents the size of a population in millions t years since 2000 and we know that

   - the birth rate is 0.05 births per person per year;
   - the death rate is 0.02 deaths per person per year;
   - 3 million immigrants join the population each year.

   Write (**but do not solve**) a differential equation for $\frac{dP}{dt}$, the rate of change of the population at time t.

millions of people

$$\frac{dP}{dt} = 0.05 \cdot P - 0.02 \cdot P + 3$$

units are millions people per year

births/person/yr

millions births per yr

millions deaths per yr

millions of immigrants per year

$$\boxed{\frac{dP}{dt} = 0.03P + 3}$$