

§5.7: Partial Fractions Decomposition

(Thanks to Faan Tone Liu)

Key Points:

- Use this method to integrate rational functions.
- Degree of numerator must be lower than degree of denominator. If needed, start with long division.
- Factor the denominator
- Solve the decomposition according to the right form:

– Linear Factors:
$$\frac{5x + 3}{(x + 1)(x + 4)} = \frac{A}{x + 1} + \frac{B}{x + 4}$$

– Repeated Linear Factors:
$$\frac{2x - 4}{(x - 2)(x + 3)^2} = \frac{A}{x - 2} + \frac{B}{x + 3} + \frac{C}{(x + 3)^2}$$

– Irreducible Quadratic Factors:
$$\frac{2x^2 - 3x - 1}{(x - 1)(x^2 + 9)} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 9}$$

– Mixtures are possible!

- Make sure you remember how to integrate the “outputs” of the partial fractions decomposition. For example:

$$\frac{5}{2x + 1}, \frac{3}{(x + 4)^2}, \frac{2x}{x^2 + 9}, \frac{4}{x^2 + 25}, \text{ or } \frac{3x + 1}{x^2 + 16}.$$

- Other notes and tips:

Examples:

1.
$$\int \frac{2x^2 + 3x - 3}{x^2 - x} dx$$

First, use long division to re-write the integrand so the fraction involved has a numerator with a lower degree than the denominator.

The form of the decomposition of $\frac{5x-3}{x(x-1)}$ is:

Here are two methods to find A and B in the partial fractions decomposition:

Method I: Equate Coefficients	Method II: Judicious Substitution

Now,
$$\int \frac{2x^2 + 3x - 3}{x^2 - x} dx =$$

$$2. \int \frac{x^2 + x - 5}{(x - 2)(x - 1)^2} dx$$

$$3. \int \frac{10}{(x+1)(x^2+9)} dx$$