§8.2: Series

(Thanks to Faan Tone Liu)

Key Points (Part I):

- An **infinite series** is the sum of the terms of a sequence:

- More precisely, an infinite series is related to a special sequence of **partial sums**:

- This allows us to see that the sum of an infinite series is:

- Graphical perspective (infinite series are related to Riemann sums):

- Important tool:

Divergence Test:

Examples:

1. (Using the divergence test) Does $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ converge or diverge?

2. (Harmonic series) Does $\sum_{n=1}^{\infty} \frac{1}{n}$ converge or diverge?

3. (Telescoping Series) Explicitly calculate the sum of the series $\sum_{i=1}^{\infty} \frac{1}{i(i+1)}$.

Key Points (PartII):

- $\sum_{n=1}^{\infty} a_n =$ ____________________.

- The goal is to determine if $\sum_{n=1}^{\infty} a_n$ converges or diverges. So far, we have a few tools:

  - **Divergence test**. Check if ____________________. If the limit is not zero, you are done and $\sum_{n=1}^{\infty} a_n$ ____________________. If $a_n \to 0$, then too bad, we have to do more.

  - We can directly calculate the partial sums $S_N = \sum_{n=1}^{N} a_n$ for **telescoping series** and take the limit $\lim_{N\to\infty} S_N$ to establish convergence or divergence.

  - **Geometric series** are our friends! A geometric series has the form __________. We know that

    * $\sum_{i=1}^{n} ar^{i-1} =$ ________________. In other words,

    * If $|r| < 1$, then $\sum_{i=1}^{\infty} ar^{i-1} =$ _______________. In other words,

    * If $|r| \geq 1$, then $\sum_{i=1}^{\infty} ar^{i-1}$ _____________.

- The **harmonic series** is ____________________. It ____________________.

- Other notes:

Examples: For each of these series, write it in expanded form if it is given in Σ-notation, and in Σ-notation if it is given in expanded form. Then, determine if the series converges and if so, find the sum.

Ex A. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{\pi}{e}\right)^{n-1}$

Ex B. $\displaystyle\sum_{i=1}^{\infty}\ln\left(\frac{i+1}{i}\right)$

Ex C. $3+\dfrac{3}{2}+1+\dfrac{3}{4}+\dfrac{3}{5}+\dfrac{3}{6}+\cdots$

Ex D. $\displaystyle\sum_{n=2}^{\infty}3\left(\frac{2}{3}\right)^{n-1}$

Ex E. $5+\sqrt{5}+\sqrt[3]{5}+\sqrt[4]{5}+\cdots$

Ex F. $1+x+x^2+x^3+\cdots$