1. Warm-up (Review of related rates from Calc 1): A cylindrical keg with radius 10 cm and height 25 cm has a drain hole at the bottom with an area of 2 cm^2. The keg is draining at a constant rate of $200\frac{\text{cm}^3}{\text{sec}}$.

   (a) Find the depth of the liquid when there are 4 L of the liquid remaining. Use the fact that 1 L = 1000 cm^3. Do you need to use calculus for this part?

   (b) How fast is the height of the liquid dropping where there are 4 L remaining in the keg? Do you need calculus for this part?

   (c) How long does it take for the tank to drain? Do you need calculus for this part?

2. According to the model of the previous problem, is the liquid coming out at a constant rate? Does this model fit with your experience?

3. Realistically, we expect the rate at which the liquid drains to depend on ________________________,

   ___________________________, and the viscosity of the liquid.

   We will soon learn to take these factors into consideration using differential equations.

4. According to Toricelli's Law, water drains from a tank according to the following law:

$$\frac{dV}{dt} = -0.6A\sqrt{2gh(t)}$$

where A is the area of the hole, g is acceleration due to gravity, and $h(t)$ is the depth of the liquid. (The constant 0.6 is based on the viscosity of water).

   (a) For the keg in problem 1, how fast is the height dropping when there are 4 L of beverage remaining?

   (b) Note that $\frac{dV}{dt}$ depends on time, as does $\frac{dh}{dt}$. Show that h satisfies the differential equation $\frac{dh}{dt} = -.1691\sqrt{h}$.

   (c) Solve the differential equation to find h. Use the initial condition that the $h = 25$ cm when $t = 0$ to solve for the constant.

   (d) Show it takes about 59.1 seconds for the keg to drain.

5. Warm-up (Review of related rates from Calc 1): A conical funnel has radius 4 cm and height 8 cm. Coffee is draining from the funnel at a constant rate of 1.3 cm^3/sec.

   (a) What is the depth of the coffee when there are 100 cm^3 of coffee remaining in the funnel? Do you need calculus for this part?

   (b) How fast is the level dropping when there are 100 cm^3 of coffee in the funnel? Do you need calculus for this part?

   (c) How long does it take for the funnel to drain? Do you need calculus for this part of the problem?

As in the first problem, it's not realistic that the funnel drains at a constant rate. We will again address this issue using differential equations.

6. Now we'll again use the more accurate model, Toricelli's law, for the rate at which the liquid is draining. Assume the area of the hole in the funnel is $A = 2$ mm^2.

$$\frac{dV}{dt} = -0.6A\sqrt{2gh(t)}$$

(a) Note that $\frac{dV}{dt}$ depends on time, as does $\frac{dh}{dt}$. Show that h satisfies the differential equation $\frac{dh}{dt} = -.68h^{-3/2}$.

(b) Solve the differential equation to find h. Use the fact the the funnel is full at time $t = 0$ to solve for the constant.

(c) How long does it take for the funnel to drain?