Goal: If we know that a power series converges to a specific function, we can manipulate the equation to determine the limits of new power series. This is a nifty and fast way to get lots of new power series representations of functions. Today we will manipulate power series in these ways:

- Substitute
- Multiply by x
- Differentiate
- Integrate

1. Write down a power series representation for the function $f(x) = \dfrac{1}{1-x}$ by using the fact that the geometric series $\displaystyle\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$. Write your answer in both expanded form and Σ-notation. On what interval does the series converge to the function?

2. Using your response for the last problem, substituting $-x$ in the place of x, find the power series representation for $f(x) = \dfrac{1}{1+x}$. Write your answer in both expanded form and Σ-notation. On what interval does the series converge to the function?

3. Find the power series representation for $f(x) = \dfrac{1}{1+x^2}$. Write your answer in both expanded form and Σ-notation. On what interval does the series converge to the function?

4. Find the power series representation for $\dfrac{x}{1-x}$. (Hint: multiply answer to problem 1 by x.) On what interval does the series converge to the function?

5. Find the power series representation for $\dfrac{1}{(1-x)^2}$. On what interval does the series converge to the function? Hint: Take the derivative of both sides of this equation:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + ... + x^n + ... = \sum_{n=0}^{\infty} x^n$$

6. Find the power series representation of $\arctan x$. (Hint: start with the power series for $\dfrac{1}{1+x^2}$ and antidifferentiate. Solve for the constant of integration by substituting $x = 0$.) On what interval does the series converge to the function?