WRITTEN HOMEWORK

All written homework is taken from the textbook,

J. Stewart, *Calculus: Concepts and Contexts*, 4th Edition,

unless otherwise indicated.

Written homework is due once a week, on Thursdays, at the **start of your recitation section**, and must be **stapled** with your **name**, **section number**, and **homework number** on it to receive credit. **Late** homework will **not** be **accepted**.

An asterisk * indicates that a homework assignment has not been finalized. *This file was last modified at 08:53 on Monday 3rd December, 2018.*

Homework 1 (due Thursday, Aug 30):

- Section 5.5: 16, 34, 38, 40, 60, 61, 66

Homework 2 (due Thursday, Sep 6):

- Section 5.6: 10, 12, 14, 22, 26, 44
- Section 5.7: 8, and the following problems:
  - **Supplement Problem A:** $\int_0^{\pi/8} \cos^4(2x)\,dx$,
  - **Supplement Problem B:** $\int \sin^3 x\,dx$,
  - **Supplement Problem C:** $\int \sec^4 x \tan^3 x\,dx$

Homework 3 (due Thursday, Sep 13):

- Section 5.7: 16, 18, 24, and
  - **Supplement Problem A:** $\int \frac{1}{\sqrt{x^2+4}}\,dx$,
  - **Supplement Problem B:** $\int \frac{dx}{x^2\sqrt{25-x^2}}$,
  - **Supplement Problem C:** $\int \frac{x^2-x+4}{x^3+4x}\,dx$
  - **Supplement Problem D:** Evaluate $\int_0^3 \sqrt{18-x^2}\,dx$ and verify your answer by interpreting the integral in terms of an area.
- Section 5.8:
  - **Supplement Problem E:** Check your answer to $\int \frac{1}{\sqrt{x^2+4}}\,dx$ on WolframAlpha. Confirm that the two answers are equivalent.
  - **Supplement Problem F:** Integrate $\int x^2(2+x^3)^4\,dx$ using the simplest method possible. Compare your answer to the result you get on Wolfram Alpha. Explain why the two answers are equivalent.
- Section 5.9: 19c, just for T_n

Homework 4 (due Thursday, Sep 20):

- Section 5.10: 14, 31, 44, 48, 57
- Section 6.1: 24, and
  - **Supplement Problem A:** Find the area of the region bounded by the graphs of $4x + y^2 = 12$ and $y = x$.
- Section 6.2: 16, 18, 20, and
  - **Supplement Problem B:** Find the volume of the solid obtained by rotating the region bounded by the graphs of $y = x^3$ and $y = \sqrt{x}$ about the line $y = 1$.
  - **Supplement Problem C:** A wedge is cut out of a circular cylinder of radius 4 by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of 30° along a diameter of the cylinder. Find the volume of the wedge.
- The following problem is *optional* (do *not* turn this in): Section 6.2: 43.

Homework 5 (due Thursday, Sep 27):

- Section 6.3: 2, 5, 6, and
  - **Supplement Problem A:** Find the volume obtained by rotating the disk bounded by the graph of the circle $x^2 + (y-1)^2 = 1$ about the line $x = 2$ by the washer method and also by the shell method.
- Section 6.5: 6, 13
- Section 6.6: 2, 10, 12

Homework 6 (due Thursday, Oct 4):

- Section 6.6: 20, 48, 50
- Section 8.1: 28, 30, 32, and
  - **Supplement Problem A:** Determine convergence/divergence of the sequence

    $$a_n = \left(1 + \frac{2}{n}\right)^{3n}.$$

    Carefully show your work.
  - **Supplement Problem B:** Determine convergence/divergence of the sequence

    $$a_n = \frac{\sqrt{3 + 9n^4}}{3n^2 + 4}.$$

    Carefully show your work.

Homework 7 (due Thursday, Oct 11):

- Section 8.2: 20, 26, 34, 42, 52 (a) and (b).
- Section 8.3: 2, 5, 12, 14, 18, 22, and
  - **Supplement Problem A:** Determine if $\sum_{n=1}^{\infty} ne^{-2n}$ converges.
  - **Supplement Problem B:** Determine if $\sum_{n=2}^{\infty} \frac{n}{\ln n}$ converges.

Homework 8 (due Thursday, Oct 18):

- Section 8.3: 10, 28, 30, 42
- Section 8.4: 4, 6, 10, 14

Homework 9 (due Thursday, Oct 25):

- Section 8.3: 32, 34
- Section 8.4: 14, 18, 22, 25, 26, 30, and
  - **Supplement Problem A:** Determine if $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+2}}$ converges absolutely, converges conditionally, or diverges. Show all details of your work.

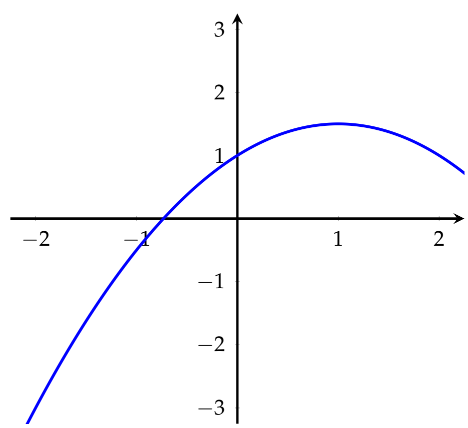
Homework 10 (due Thursday, Nov 1):

- Do the following problems on Taylor Polynomials:
  - **Supplement Problem A:** Do, but don't turn in: Memorize the formula for the nth-degree Taylor Polynomial for $f(x)$ centered at a:

$$T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(x)}{n!}(x-a)^n$$
$$= \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i$$

  - **Supplement Problem B:** Find the 4th degree Taylor polynomial for $\tan(x)$ centered at $a = 0$.
  - **Supplement Problem C:** Suppose that a function $f(x)$ is approximated near $a = 0$ by the 3rd degree Taylor polynomial $T_3(x) = 4 - 3x + \frac{x^2}{5} + 4x^3$. Give the values of $f(0), f'(x), f''(0)$, and $f'''(0)$.
  - **Supplement Problem D:** Find the 10th degree Taylor polynomial centered at $a = 1$ of the function $f(x) = 2x^2 - x + 1$.
  - **Supplement Problem E:** Suppose a function $f(x)$ has the following graph:

Graph of $f(x)$



If the 2nd degree Taylor polynomial centered at $a = 0$ for $f(x)$ is $T_2(x) = bx^2 + cx + d$, determine the signs of b, c and d.

  - **Supplement Problem F:** Show your work in an organized way:
    1. Find the 7th degree Taylor polynomial centered at $a = 0$ for $\sin(x)$.
    2. Use $T_7(x)$ to estimate $\sin(3^\circ)$. Don't forget to convert to radians.
    3. Compare your estimate for $\sin(3^\circ)$ to the value that technology gives you. How accurate is the approximation you found?

- Section 8.5: 12, 20, 22, 26
- Section 8.6: 8, 12, 30

Homework 11 (due Thursday, Nov 8):

- Section 8.7: 20 (show $\lim_{n\to\infty} R_n(x) = 0$ for all x)
- Section 8.7: 28, 30, 40 (five decimal places means to within 0.000005, and include a careful explanation of how you chose n)
- Section 8.7: 50, 64
- Section 8.8: 14, 22, 24

The following homework problems are optional, and **NOT** to be turned in:

- Do the following problems on Taylor Polynomials:
  - **Supplement Problem A:** This problem asks for Taylor polynomials for $f(x) = \ln(1+x)$ centered at $a = 0$. Show your work in an organized way.
    1. Find the 4th, 5th, and 6th degree Taylor polynomials for $f(x) = \ln(1+x)$ centered at $a = 0$.
    2. Find the nth degree Taylor polynomial for $f(x)$ centered at $a = 0$, **written in expanded form**.
    3. Find the nth degree Taylor polynomial for $f(x)$ centered at $a = 0$, **written in sigma (summation) notation**.
    4. Use the 7th degree Taylor polynomial to estimate $\ln(2)$.
    5. Compare your answer to the estimate for $\ln(2)$ given by your calculator. How accurate were you?
    6. Looking at the Taylor polynomials, explain why this estimate is less accurate than the estimate in the previous problem for $\sin(3^\circ)$
  - **Supplement Problem B:** Do, but don't turn in: memorize the nth degree Taylor polynomials centered at $a = 0$ for e^x, $\sin(x)$, $\cos(x)$, $\ln(1+x)$, and $\frac{1}{1-x}$. Be able to write them down with ease in both expanded form and sigma notation.

- Section 8.8:
  - **Supplement Problem C:** Find the Taylor series for xe^x about x=0. Then integrate term-by-term and substitute to show that the series $\sum_{n=0}^{\infty} \frac{1}{n!(n+2)}$ converges to 1.
  - **Supplement Problem D:** We wish to estimate $\ln 0.5$ using an nth degree Taylor polynomial for $\ln(1+x)$ centered at $a = 0$. How large should n be to guarantee the approximation will be within 0.0001? (Hint: start by calculating a formula for $|(f^{(n+1)}(x)|$ and finding a bound for it between $-1/2$ and 0.) (If you need some help, there is a sample Taylor inequality problem, with solutions, in the Calculus 2 section of the Resources webpage, linked on the left side of this webpage.)

Homework 12 (due Thursday, Nov 15):

- Section 7.1: 4, 10
- Section 7.2: 8, 10, 22
- Section 7.3: 6, 4, 14, and review related rates (Section 4.1; see also the Calculus 1 Review section of the resources webpage for the course, and the Related Rates Review there).
- Then more practice to do (but do not turn in): Section 7.2: 12, 18.

The following homework problems are optional, and **NOT** to be turned in:

- Section 7.3: 20
- Start Project 12 on Toricelli's Law:

`http://math.colorado.edu/~casa/teaching/18fall/2300/schedule/2300schedule_Fall.html`

Homework 13 (due Thursday, Nov 29):

- Section 7.3: 34
- Section 7.4: 10, 14, 18
- Section 7.5: 10

The following homework problems are optional, and **NOT** to be turned in:

- Section 7.3:
  - **Supplement Problem A:** A certain small country has \$10 billion in paper currency in circulation. Each day \$50 million of the money in circulation enters the country's banks, and another \$50 million leaves the banks and enters circulation. The government decides to introduce new currency by having the banks replace the old bills with the new ones whenever old currency comes into the banks. Let $x = x(t)$ denote the amount of new currency in circulation at time t, with $x(0) = 0$. Assume that the proportion of new money entering the banks each day is the same as the proportion of new money in circulation. How long would you estimate it to take for the new bills to account for 90% of the currency in circulation?

Homework 14 (due Thursday, Dec 6):

- Section 1.7: 18, 20 (on both of these, eliminate the parameter to produce the graph. Show on the graph the start/end points and the direction of motion)
- Section 3.4: 84 (Use technology to produce the graph)
- Section 6.4: 3, 14, 26

The following homework problems are optional, and **NOT** to be turned in:

- Problems 19-24 on Page 700 of the textbook, and
  - **Supplement Problem A:** Find a parameterization for the line segment going from the point $(2, 3, -4)$ to the point $(5, -2, -4)$.
- Section 3.4:
  - **Supplement Problem B:** For the parametric curve $x = t^3 + t$, $y = t^2$, find the equation of the tangent line at $t = 1$, find the speed at $t = 1$, and then find $\frac{d^2y}{dx^2}$ at $t = 1$ to determine if the curve is concave up or concave down there.

Homework 15 (due Thursday, Dec 13):

- Appendix H.1: 14, 34, 36, 54
- Appendix H.2: 16, 22

The following homework problems are optional, and **NOT** to be turned in:

- Appendix H.2:
  - **Supplement Problem A:** Find the area of the region that lies inside both polar curves $r = 1 + \cos(\theta)$ and $r = 1 - \cos(\theta)$.
  - **Supplement Problem B:** Find the exact length of the polar curve $r = e^{2\theta}$, $0 \leq \theta \leq 2\pi$.