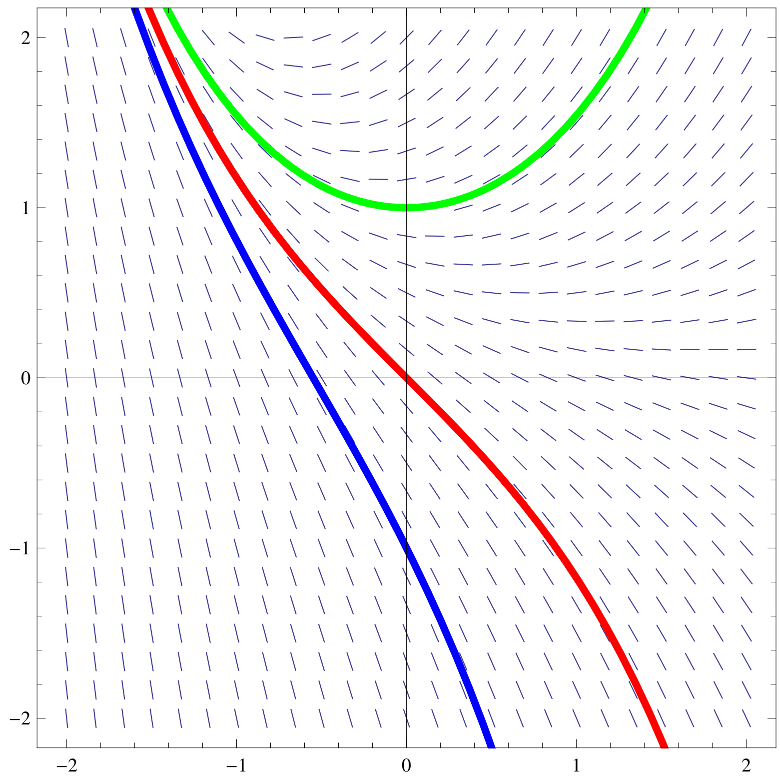
MATH 2300 – CALCULUS II – UNIVERSITY OF COLORADO

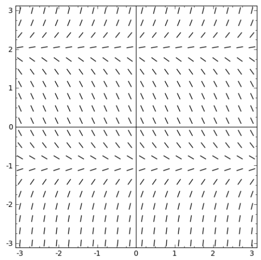
Final exam review problems

1. A slope field for the differential equation $y' = y - e^{-x}$ is shown. Sketch the graphs of the solutions that satisfy the given initial conditions. Make sure to label each sketched graph.

   (a) $y(0) = 0$ *RED* (b) $y(0) = 1$ *GREEN* (c) $y(0) = -1$ *BLUE*



2. For each differential equation, find the corresponding slope field. (*Not all slope fields will be used.*)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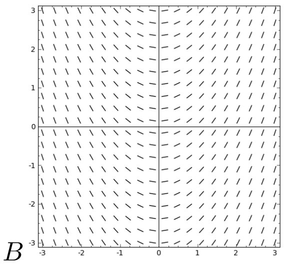
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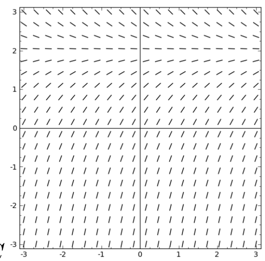
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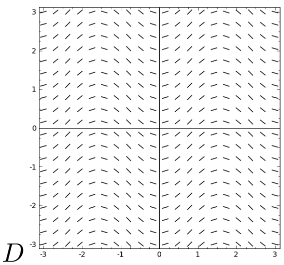
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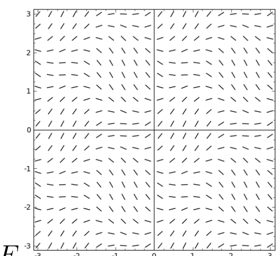
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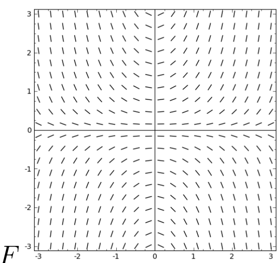

F

Equation	**Slope Field**
$\frac{dy}{dx} = x$	B
$y' = xy$	F
$y' = \sin(2x)$	D
$\frac{dy}{dx} = 2 - y$	C

3. For the previous problem, which slope fields have equilibrium solutions? Are they stable or unstable?

   A has an unstable equilibrium solution at $y = 2$ and a stable solution at $y = -1$. C has a stable equilibrium at $y = 2$ (which can be verified directly from its corresponding differential equation). F has an unstable equilibrium at $y = 0$ (which also can be determined from its corresponding differential equation.)

4. A bacteria culture contains 300 cells initially and grows at a rate proportional to its size. After half an hour the population has increased to 540 cells.

   (a) Write down the differential equation describing the situation.

   Let $P(t)$ be the bacteria population size (the number of bacteria) at time t (in hours). Since the bacteria grows at a rate ($\frac{dP}{dt}$) **proportional** to its size P, we have the following DE modeling the problem,

   $$\frac{dP}{dt} = kP, \quad \text{where } k > 0 \text{ is a constant.}$$

   (b) Solve the differential equation.

   The differential equation can be solved by separating variables as follows,

   $$\begin{aligned} \frac{1}{P} dP &= k\,dt && \text{(separate } P\text{'s and } t\text{'s)} \\ \int \frac{1}{P} dP &= \int k\,dt && \text{(integrate both sides)} \\ \ln|P| &= kt + C_1 && \text{(solving it for } P \text{ gives)} \\ P(t) &= Ce^{kt} && (C = e^{C_1} \text{ is an arbitrary constant.)} \end{aligned}$$

(c) Use the remaining information in the problem to solve for the two constants, and write down the model for the number of bacteria at time t.

From (b), $P(t) = Ce^{kt}$, where C and k are two constants to be determined.
"A bacteria culture contains 300 cells initially" $\Longrightarrow P(0) = 300$ cells.
On the other hand, as $t = 0$, $P(0) = Ce^{k\cdot 0} = C$, So $C = 300$, and $P(t) = 300e^{kt}$.
"After half an hour the population has increased to 540 cells" $\Longrightarrow P(0.5) = 540$ cells
So we have $300e^{k\cdot 0.5} = 540$. Dividing both sides by 300 yields $e^{0.5k} = \dfrac{540}{300}$, or $e^{\frac{k}{2}} = \dfrac{9}{5}$.
$\dfrac{k}{2} = \ln\left(\dfrac{9}{5}\right) \Longrightarrow k = 2\ln\left(\dfrac{9}{5}\right) \approx 1.17557$.
So the model for the number of bacteria at time t is $P(t) = 300e^{1.17557t}$.
Note – solution 2: From $e^{\frac{k}{2}} = \frac{9}{5}$, we have $e^k = \left(\frac{9}{5}\right)^2$, and then
$P(t) = 300e^{kt} = 300\left(e^k\right)^t = 300\left(\left(\frac{9}{5}\right)^2\right)^t = 300\left(\frac{9}{5}\right)^{2t}$. Or $P(t) = 300\left(\frac{9}{5}\right)^{2t} = 300e^{1.17557t}$.

(d) How long until there are 10000 bacteria?

Solve $P(t) = 10000$ for t, or solve $300e^{1.17557t} = 10000$ for t.
$e^{1.17557t} = \dfrac{100}{3}$, $t = \dfrac{\ln\left(\frac{100}{3}\right)}{1.17557} \approx 2.98285$ (hours).

5. Cobalt-60 has a half life of 5.24 years. We begin with a 50 mg sample.

(a) Write down the applicable differential equation, given that radioactive substances decay at a rate proportional to the remaining mass.

Denote $m(t)$ the remaining mass of Cobalt-60 at time t years. Since "the decay rate is **proportional** to the remaining mass, we have

$$\frac{dm}{dt} = km, \quad \text{where } k < 0 \text{ is a constant.}$$

(b) Solve the differential equation, using the other information in the problem to determine the constants. Write down the model for the amount of Cobalt-60 remaining after t years.

By separating variables, we have

$$\begin{aligned} \frac{1}{m}\,dm &= k\,dt && \text{separate } m\text{'s and } t\text{'s} \\ \int \frac{1}{m}\,dm &= \int k\,dt && \text{integrate both sides} \\ \ln|m| &= kt + C_1 && \text{solve it for } m \\ m &= Ce^{kt} && C = e^{C_1} \text{ is an arbitrary constant.} \end{aligned}$$

To determine the two constants C and k, note that "we begin with a 50 mg sample", and so $m(0) = 50$, and therefore $m(0) = Ce^{k\cdot 0} = C$, or $C = 50$. Since "Cobalt-60 has a half life of 5.24 years", $m(5.24) = \frac{1}{2}m(0)$, or $50e^{5.24k} = \dfrac{1}{2}\cdot 50$. Solving it for k,

$$\frac{1}{2} = e^{5.24k} \Leftrightarrow 5.24k = \ln\left(\frac{1}{2}\right) \Leftrightarrow k = \frac{-\ln 2}{5.24} \approx -0.13228$$

So the model for the amount of Cobalt-60 remaining after t years is $m(t) = 50e^{-0.13228t}$.

(c) How much remains after 20 years? How many years until 1 mg remains?

After 20 years, $m(20) = 50e^{-0.13228\cdot 20} = 3.54814$ mg of Cobalt-60 remains.
To find many years until 1 mg remains, we solve $m(t) = 1$ mg for t, or solve $50e^{-0.13228t} = 1$ for t.

$$e^{-0.13228t} = \frac{1}{50} \Leftrightarrow -0.13228t = \ln\left(\frac{1}{50}\right) \Leftrightarrow t = \frac{\ln\left(\frac{1}{50}\right)}{-0.13228} = \frac{-\ln(50)}{-0.13228} \approx 29.5738 \ \text{(years)}$$

6. The rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. Write down the differential equation that models this statement, and solve it. Now a turkey at a room temperature of $70°F$ is put into a 350°F oven at 11:00 am. 1 hour later the meat has a temperature of 100°F. I serve my cooked turkey at 160°F. If I need to remove the perfectly cooked turkey from the oven at 3:00, should I turn the oven temperature up or down? (Note: The temperatures given in this problem should not be considered official safety information. Consult the Food And Drug Agency's website for health recommendations for the cooking of poultry.)

   We denote $T(t)$ the temperature of the turkey at time t, and T_s the temperature of the surrounding environment, which is equal to $350°F$. Since "the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings", we have, by **Newton's Law of Cooling**,

   $$\frac{dT}{dt} = k(T - T_s), \quad \text{where } k \text{ is the proportionality constant.}$$

   We solve the above differential equation by separating variables,

   $$\begin{aligned}
   \frac{dT}{T - T_s} &= k\,dt && \text{(separate } T \text{ and } t\text{)}\\
   \int \frac{dT}{T - T_s} &= \int k\,dt && \text{(integrate both sides)}\\
   \ln|T - T_s| &= kt + C_1 && \text{(solve for } T\text{)}\\
   T(t) &= Ce^{kt} + T_s && (C = e^{C_1} \text{ is a constant.})
   \end{aligned}$$

   By the given information, we have $T(0) = 70°F$ (turkey's initial temperature). So we get $T(0) = Ce^{k\cdot 0} + 350 = 70 \Longrightarrow C \cdot e^0 + 350 = 70 \Longrightarrow C = 70 - 350 = -280$. So $T(t) = -280e^{kt} + 350$. Note 1 hour after the turkey was put into even, it has a temerature of $100°F$. So $T(1) = 100°F$, or

   $$-280e^{k\cdot 1} + 350 = 100 \Longleftrightarrow -280e^k = 100 - 350 \Longleftrightarrow e^k = \frac{25}{28} \Longleftrightarrow k = \ln\left(\frac{25}{28}\right) \approx -0.113329$$

   So the model for temperature of turkey at time t is $T(t) = -280e^{-0.113329t} + 350$.

   In order to answer the last question, we need to find the time t when turkey's temperature reaches $160°F$.

   $$160 = -280e^{-0.113329t} + 350 \Longleftrightarrow e^{-0.113329t} = \frac{160 - 350}{-280} = \frac{19}{28} \Longleftrightarrow t = \frac{\ln\left(\frac{19}{28}\right)}{-0.113329} \approx 3.4216$$

   So 3.4216 hours later from 11:00 am, i.e., at around 2:25 PM, the turkey will be done (temperature will be $160°F$), and so I should turn oven temperature down.

7. How long will it take an investment to double if it is invested at 6% interested, compounded continuously? What about at 3%? Note that growth rate of an investment with continuously compounded interested are proportional to the current size of the investment.

   Let $A(t)$ be the amount of investment at time t, and $A_0 = A(0)$. Since the growth rate of an investment with continuously compounded interests is proportional to its size $A(t)$, $A'(t) = k\,A(t)$, so $A(t) = A_0e^{kt}$, where k is the interest rate.

   (a) To find how long it will take an investment to double if the rate is 6%, we solve $2A_0 = A_0e^{0.06t}$ for t.
   $2 = e^{0.06t} \Longleftrightarrow 0.06t = \ln(2) \Longleftrightarrow t = \dfrac{\ln(2)}{0.06} \approx 11.5525$ (years).

   (b) To find how long it will take an investment to double if the rate is 3%, we solve $2A_0 = A_0e^{0.03t}$ for t.
   $2 = e^{0.03t} \Longleftrightarrow 0.03t = \ln(2) \Longleftrightarrow t = \dfrac{\ln(2)}{0.03} \approx 23.1049$ (years).

8. The world population was estimated to be 190 million in the year 400 CE. Assuming a logistic growth model with a carrying capacity of 15 billion, and a growth rate of about 0.2% per year when the population was very small, write down the differential equation that models this situation.

   Let $P(t)$ be the world population in million at time t after the year 400 CE (corresponding to $t = 0$). We assume the world population follows the logistic growth model with a carrying capacity of $M = 15$ billion $= 15,000$ million, and a relative growth rate $k = 0.2\% = 0.002$. So we have the following initial value problem that models this situation.

$$\begin{cases} \frac{dP}{dt} & = & 0.002P\left(1 - \frac{P}{15{,}000}\right) \\ P(0) & = & 190 \end{cases}$$

9. The air in a room with volume 180 m^3 contains 0.15% carbon dioxide initially. Fresher air with only 0.05% carbon dioxide flows into the room at a rate of 2 m^3/min and the mixed air flows out at the same rate. Write down the initial value problem modeling this situation.

   Let $P(t)$, in m^3, be the percentage of carbon dioxide (CO_2) at time t (in minutes). Then $P(0) = 0.15$ (the room contains 0.15% CO_2 initially.).

$$\text{rate-in} = (0.05)(2\ m^3/\text{min}) = 0.1m^3/\text{min}$$
$$\text{rate-out} = P(t)(2\ m^3/\text{min}) = 2P(t)m^3/\text{min}$$

   Thus

$$\frac{dP}{dt} = (\text{rate-in}) - (\text{rate-out}) = 0.1 - 2P$$

   So the initial value problem modeling this situation is $\begin{cases} \frac{dP}{dt} & = & 0.1 - 2P \\ P(0) & = & 0.15 \end{cases}$

   Note: This IVP can be solved by separating variables,

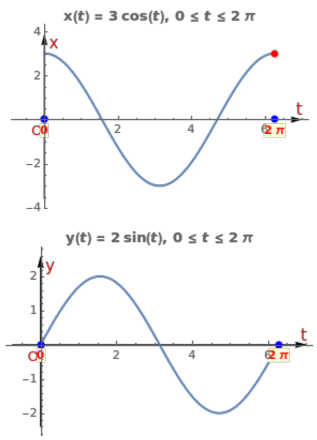
$$\frac{dP}{0.1 - 2P} = dt \iff -\ln|0.1 - 2P| = t + C_1 \iff 0.1 - 2P = \pm e^{-t-C_1}$$
$$\iff P = \frac{0.1}{2} \pm \frac{1}{2}e^{-t-C_1} = 0.05 \pm \frac{1}{2}e^{-C_1}e^{-t}$$
$$\iff P(t) = 0.05 + Ce^{-t} \qquad (\text{where } C = \pm\frac{1}{2}e^{-C_1} \text{ is a constant.})$$

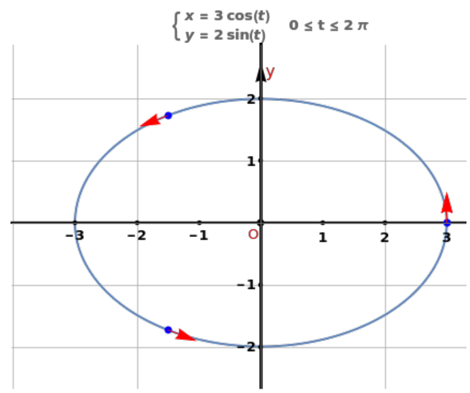
   Since $P(0) = 0.15$, $0.15 = 0.05 + Ce^0$, and $C = 0.1$. This gives $P(t) = 0.05 + 0.1e^{-t}$.

10. Graph the curve traced by each of the following parametric equations:

(a) $x = 3\cos t, \qquad y = 2\sin t, \qquad 0 \le t \le 2\pi$

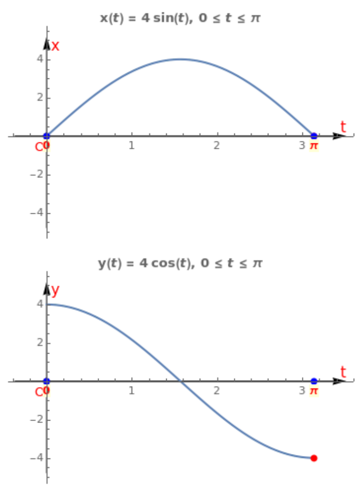
As $t = 0$, $x = 3\cos 0 = 3$, $y = 2\sin 0 = 0$; as $t = 2\pi$, $x = 3\cos(2\pi) = 3$, $y = 2\sin(2\pi) = 0$. The curve starts at $(3, 0)$ and rotates counter-clockwise and then go back to $(3, 0)$ for $t \in [0, 2\pi]$.

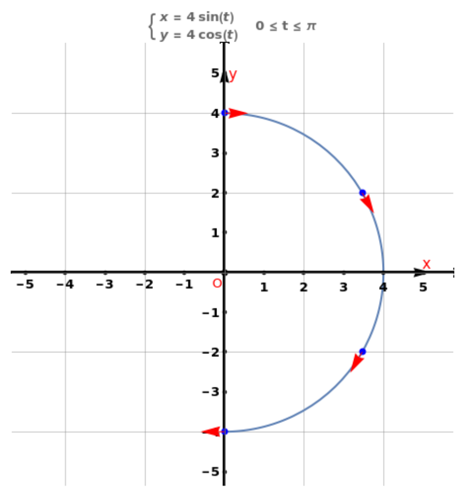




(b) $x = 4\sin t, \qquad y = 4\cos t, \qquad 0 \le t \le \pi$

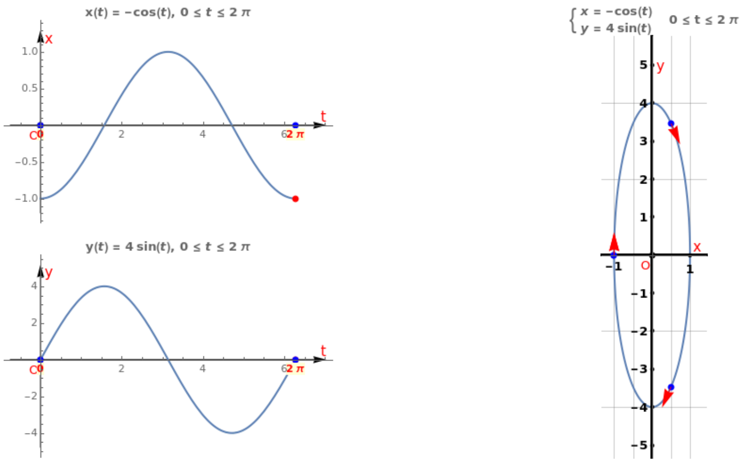
The curve starts from $(0, 4)$ $(t = 0)$ and ends at $(0, -4)$ $(t = \pi)$.





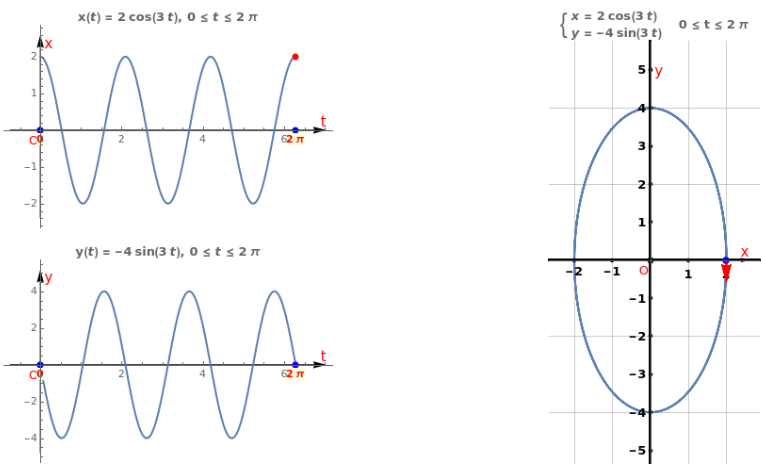
(c) $x = -\cos t, \qquad y = 4\sin t, \qquad 0 \le t \le 2\pi$

The curve starts from $(-1, 0)$ $(t = 0)$ and ends at the same point $(-1, 0)$ $(t = 2\pi)$.



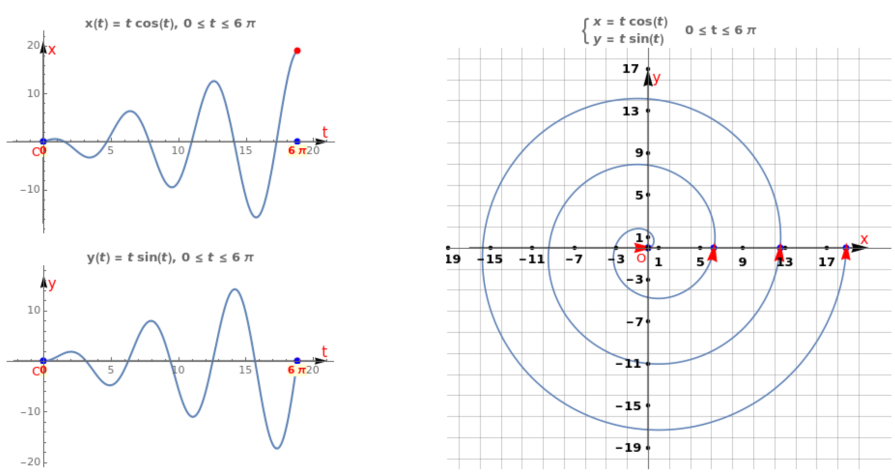
(d) $x = 2\cos 3t, \qquad y = -4\sin 3t, \qquad 0 \le t \le 2\pi$

The curve starts from $(2, 0)$ $(t = 0)$ and ends at the same point $(2, 0)$ $(t = 2\pi)$.



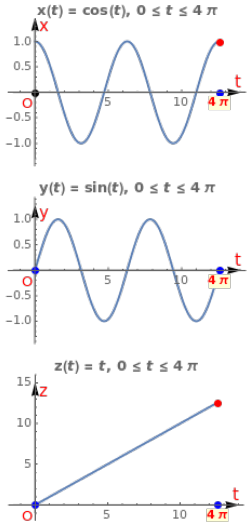
(e) $x = t\cos t, \qquad y = t\sin t, \qquad 0 \le t \le 6\pi$

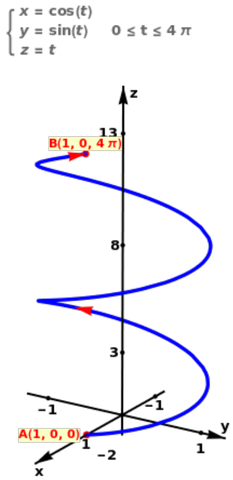
The curve starts from $(0, 0)$ $(t = 0)$ and ends at $(6\pi, 0)$ $(t = 6\pi)$.



(f) $x = \cos t, \qquad y = \sin t,\ z = t, \qquad 0 \le t \le 4\pi$

The curve starts from $A(1, 0, 0)$ $(t = 0)$ and ends at $(1, 0, 4\pi)$ $(t = 4\pi)$.

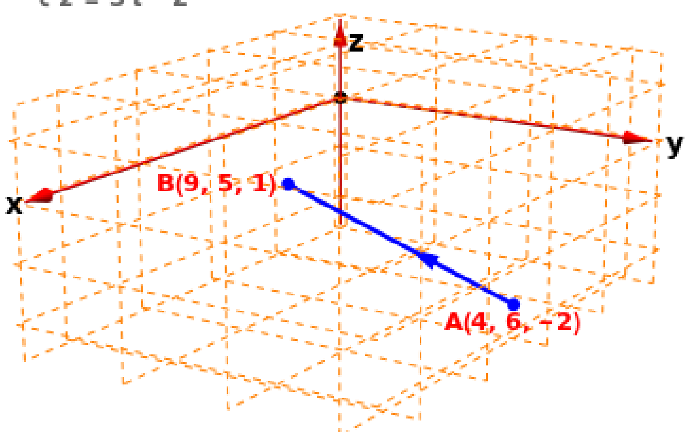




(g) $x = 4 + 5t, \qquad y = 6 - t,\ z = -2 + 3t, \qquad 0 \le t \le 1$

This is a line segment from $A(4, 6, -2)$ to $B(9, 5, 1)$.

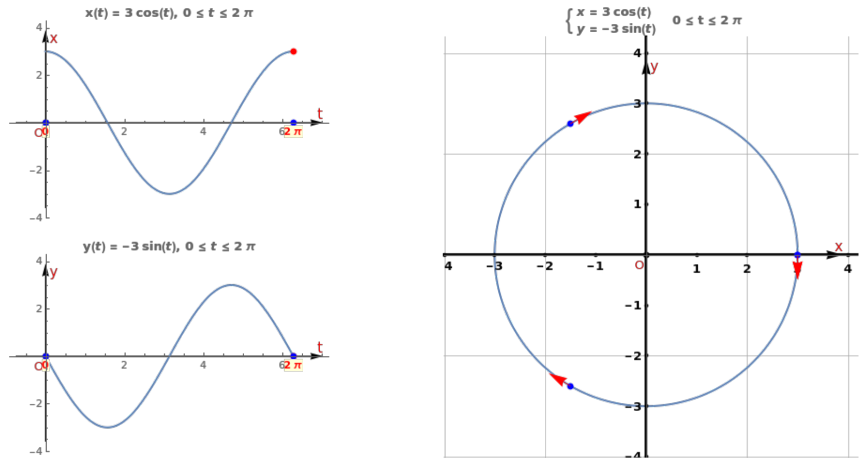
$$\begin{cases} x = 5t + 4 \\ y = 6 - t \\ z = 3t - 2 \end{cases} \quad 0 \le t \le 1, \quad \begin{cases} t = 0 \Rightarrow A(4, 6, -2) \\ t = 1 \Rightarrow B(9, 5, 1) \end{cases}$$



11. Create a parameterization for the described curves.

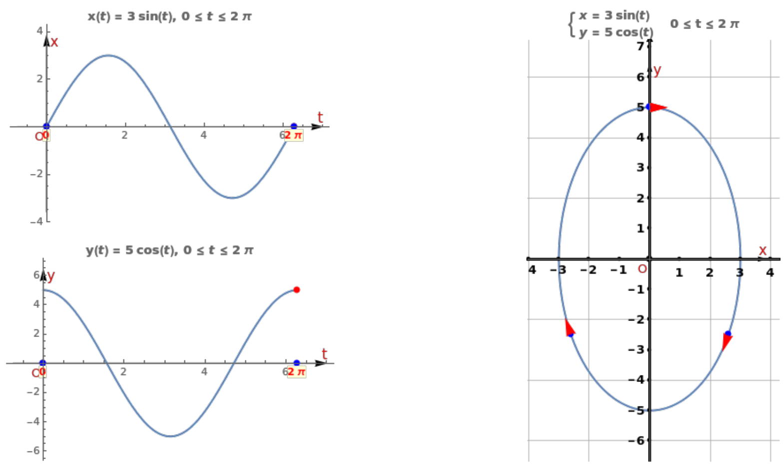
(a) A circle starting at the point $(3, 0)$, traversing a circle centered at the origin, moving clockwise, traveling once around the circle.

$x = 3\cos t$, $y = -3\sin t$, $0 \le t \le 2\pi$.

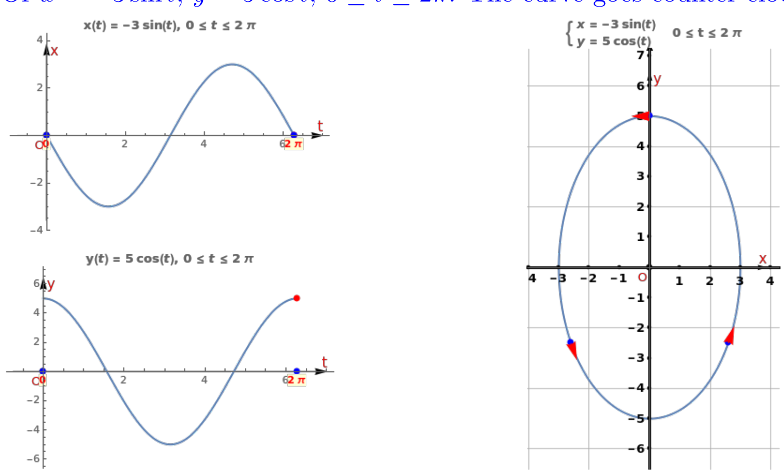


(b) Traverse the ellipse $x^2/9 + y^2/25 = 1$, start on the positive y-axis. Go in either direction, but state which direction your parameterization traverses.

$x = 3\sin t$, $y = 5\cos t$, $0 \le t \le 2\pi$. This will go clockwise starts at $(0, 5)$.



Or $x = -3\sin t$, $y = 5\cos t$, $0 \le t \le 2\pi$. The curve goes counter-clockwise from $(0, 5)$.



(c) A line segment starting at $(2, 3)$ and ending at $(-3, 5)$.
$x = 2 - 5t$, $y = 3 + 2t$, $0 \le t \le 1$ (see figure below).
Note: General form of the line segment is $x = x_0 + (x_1 - x_0)t$, $y = y_0 + (y_1 - y_0)t$, $0 \le t \le 1$, where (x_0, y_0) is the initial point, and (x_1, y_1) is the terminal point.

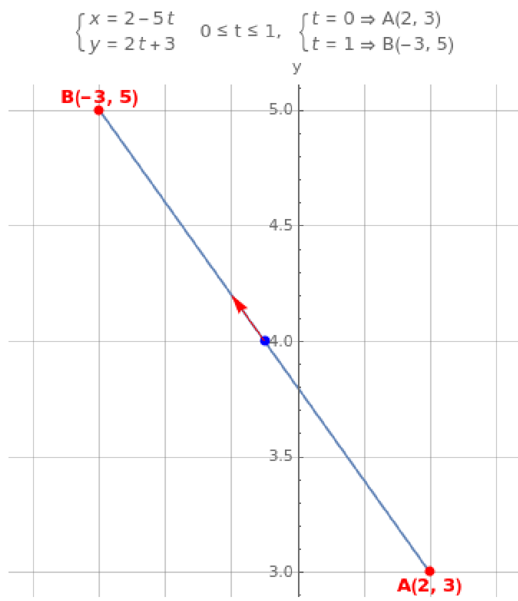


Figure for Problem 11c

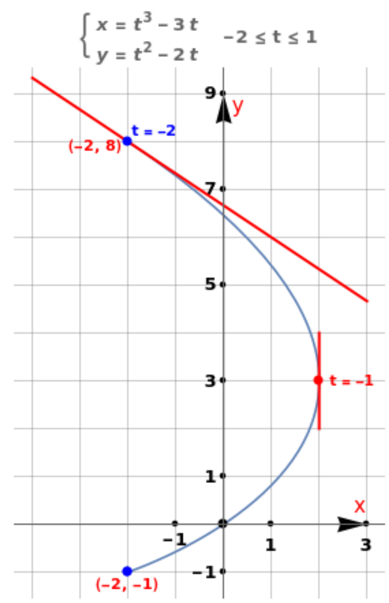


Figure for Problem 12

12. For the parameterized curve $x = t^3 - 3t, \qquad y = t^2 - 2t$:

(a) Find the equation of the tangent line to the curve at $t = -2$.
The slope of the tangent line to the curve at $t = -2$ is $\left.\dfrac{dy}{dx}\right|_{t=-2} = \left.\dfrac{y'(t)}{x'(t)}\right|_{t=-2}$.

$$\frac{dy}{dt} = \frac{d}{dt}\left(t^2 - 2t\right) = 2t - 2, \quad \frac{dx}{dt} = \frac{d}{dt}\left(t^3 - 3t\right) = 3t^2 - 3.$$
$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{2t-2}{3t^2-3} = \frac{2(t-1)}{3(t+1)(t-1)} = \frac{2}{3(t+1)}$$

So the slope at $t = -2$ is $k = \left.\frac{dy}{dx}\right|_{t=-2} = \left.\frac{y'(t)}{x'(t)}\right|_{t=-2} = \left.\frac{2}{3(t+1)}\right|_{t=-2} = \frac{2}{3(-2+1)} = -\frac{2}{3}$.
The Cartesian coordinates of the point on the curve at $t = -2$ are
$x(-2) = (-2)^3 - 3(-2) = -8 + 6 = -2$, $y(-2) = (-2)^2 - 2(-2) = 4 + 4 = 8$.
So the equation of the tangent line at $t = -2$ is (see the figure on the right above this problem.),

$$y - 8 = -\frac{2}{3}\left(x - (-2)\right) \Longleftrightarrow y = 8 - \frac{2}{3}(x + 2) \Longleftrightarrow y = \frac{2}{3}(10 - x)$$

(b) Find d^2y/dx^2 at $t = -2$.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{2}{3(t+1)}\right)}{3t^2-3} = \frac{-\frac{2}{3(t+1)^2}}{3t^2-3} = \frac{-2}{9(t+1)^3(t-1)}$$

So $\left.\dfrac{d^2y}{dx^2}\right|_{t=-2} = \left.\dfrac{-2}{9(t+1)^3(t-1)}\right|_{t=-2} = \dfrac{-2}{9[(-2)+1]^3[(-2)-1]} = -\dfrac{2}{27}$.

(c) Find the speed at $t = -2$.

The speed at $t=-2$ is $|\mathbf{r}'(t)| = \sqrt{[x'(t)]^2 + [y'(t)]^2} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ evaluated at $t=-2$.

$$\begin{aligned} \text{speed} &= \sqrt{\left(3t^2-3\right)^2 + \left(2t-2\right)^2}\,\bigg|_{t=-2} = \sqrt{(12-3)^2 + (-4-2)^2} \\ &= \sqrt{81+36} = 3\sqrt{13}. \end{aligned}$$

(d) Is the tangent line to the curve ever vertical? When and where?

Note $\dfrac{dy}{dx} = \dfrac{2}{3(t+1)}$. The tangent line is vertical if its slope $\frac{dy}{dx}$ is ∞, or if $t=-1$.

(e) Is the tangent line to the curve ever horizontal? When and where?

Since $\dfrac{dy}{dx} = \dfrac{2}{3(t+1)}$ can never be zero for any $t \in \mathbb{R}$. So there are no horizontal tangent lines.

(f) Does the particle ever come to a stop?

The particle stops if both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are zero at the same time t. So we solve $\frac{dx}{dt} = 0$, or $3t^2 - 3 = 0$, and we have $t = \pm 1$. Solving $\frac{dy}{dt} = 0$, or $2t - 2 = 0$ gives $t = 1$. So the particle stops at $t = 1$.

(g) Write down the integral giving arc length from $t=-2$ to $t=1$.

The arc length from $t=-2$ to $t=1$ is

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_{-2}^{1} \sqrt{(3t^2-3)^2 + (2t-2)^2}\, dt$$

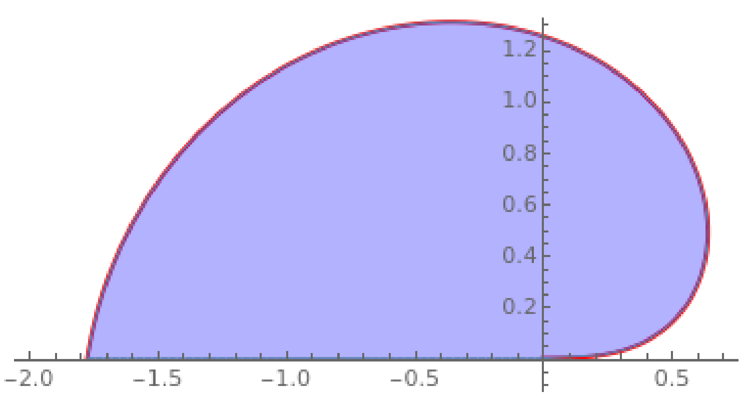
(h) What is the average speed from $t=-2$ to $t=1$?

Note speed $= \frac{\text{distrance traveled}}{\text{time elapsed}}$. The average speed from $t=-2$ to $t=1$ is the average value of the function $\sqrt{(3t^2-3)^2 + (2t-2)^2}$, which can be computed as

$$\frac{1}{b-a}\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \frac{1}{1-(-2)}\int_{-2}^{1} \sqrt{(3t^2-3)^2 + (2t-2)^2}\, dt \approx 4.21286$$

13. Find the area of the region bounded by the curve $r = \sqrt{\theta}$, for $0 \le \theta \le \pi$, and $\theta = \pi$.

$$\text{area} = \int_0^{\pi} \frac{1}{2}[r(\theta)]^2\, d\theta = \int_0^{\pi} \frac{(\sqrt{\theta})^2}{2}\, d\theta = \frac{\pi^2}{4}$$



14. Find the area of the region that lies within the limaçon $r = 3 + 2\cos(\theta)$ and outside the circle $r = 4$.

Solving $r = 3 + 2\cos(\theta)$ and $r = 4$, we get $\cos(\theta) = \frac{1}{2}$, and so for $\theta \in [-\pi, \pi]$, the solutions are $\theta = \pm\frac{\pi}{3}$. So the area bounded by the two curves is between the two rays $\theta = \pm\frac{\pi}{3}$ (see figure below on the left).

$$\text{area} = \frac{1}{2}\int_{-\pi/3}^{\pi/3} [(3 + 2\cos\theta)^2 - 4^2]\, d\theta = \int_{-\pi/3}^{\pi/3} \left(2\cos^2\theta + 6\cos\theta - \frac{7}{2}\right) d\theta = \int_{-\pi/3}^{\pi/3} \left[\cos(2\theta) + 6\cos\theta - \frac{5}{2}\right] d\theta$$

$$= \left[\frac{1}{2}\sin(2\theta) + 6\sin\theta - \frac{5\theta}{2}\right]_{-\pi/3}^{\pi/3} = \frac{13\sqrt{3}}{2} - \frac{5\pi}{3} \quad \left[\text{We are using: } \cos^2\theta = \frac{1 + \cos(2\theta)}{2}\right]$$

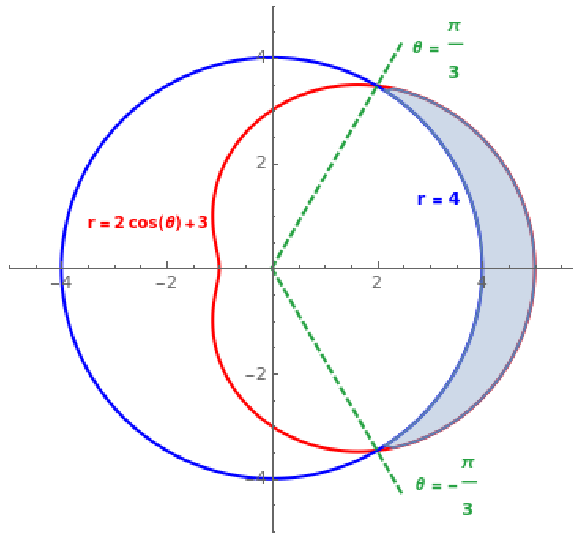


Figure for Problem 14

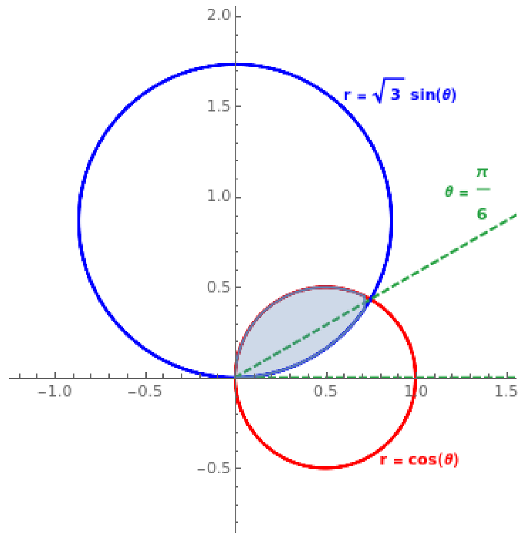


Figure for Problem 15

15. Find the area of the region common to the circles $r = \cos(\theta)$ and $r = \sqrt{3}\sin(\theta)$. [Hint: $\tan^{-1}(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}$.]

Solving $r = \cos(\theta)$ and $r = \sqrt{3}\sin(\theta)$ gives $\cos(\theta) = \sqrt{3}\sin(\theta)$, or $\cot(\theta) = \sqrt{3}$, so $\theta = \frac{\pi}{6}$. The region can be split by the line $\theta = \frac{\pi}{6}$ into two parts (see the figure above on the right):

$$\text{area} = \frac{1}{2}\int_0^{\pi/6} (\sqrt{3}\sin\theta)^2\, d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} (\cos\theta)^2\, d\theta = \frac{1}{24}\left(5\pi - 6\sqrt{3}\right) \approx 0.221486$$

16. Find the exact length of the polar curve $r = e^\theta$, for $0 \le \theta \le \pi/2$.

$$x(\theta) = r\cos\theta = e^\theta\cos\theta, \quad y(\theta) = r\sin\theta = e^\theta\sin\theta$$

$$x'(\theta) = \frac{d}{d\theta}\left(e^\theta\cos\theta\right) = e^\theta\cos\theta - e^\theta\sin\theta = e^\theta\left(\cos\theta - \sin\theta\right)$$

$$y'(\theta) = \frac{d}{d\theta}\left(e^\theta\sin\theta\right) = e^\theta\sin\theta + e^\theta\cos\theta = e^\theta\left(\cos\theta + \sin\theta\right)$$

$$x'(\theta)^2 = e^{2\theta}\left(\cos\theta - \sin\theta\right)^2 = e^{2\theta}\left(\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta\right)$$

$$y'(\theta)^2 = e^{2\theta}\left(\cos\theta + \sin\theta\right)^2 = e^{2\theta}\left(\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta\right)$$

$$L = \int_0^{\pi/2} \sqrt{x'(\theta)^2 + y'(\theta)^2}\, d\theta = \int_0^{\pi/2} \sqrt{2e^{2\theta}(\cos^2\theta + \sin^2\theta)}\, d\theta$$

$$= \sqrt{2}\int_0^{\pi/2} e^\theta\, d\theta = \sqrt{2}(e^{\pi/2} - 1)$$