

Math 2300

Exam 1 topics

The exam will cover sections 5.6, 5.7, trig integrals, partial fractions, 5.9, 5.10, 6.1, 6.2 and 6.3. Here is a list of topics that you should have mastered for the exam:

- Know the integrals of trigonometric functions, such as $\cos(x)$, $\sin(x)$, $\sec^2(x)$, $\sec(x)\tan(x)$, $\frac{1}{x}$, e^x , a^x , x^r , $\frac{1}{1+x^2}$, $\frac{1}{\sqrt{1-x^2}}$
- Be able to recognize when an algebraic simplification will make an integral easier.
- Be able to recognize integrals that are most easily done by a u/du substitution, and be able to perform these substitutions.
- Be able to evaluate a definite integral when it is done with a u/du substitution (i.e., handle the limits of integration correctly).
- Recognize when an integral is best done with an integration by parts, be able to make an effective choice for both u and v' . Be able to perform the integration by parts correctly. Be able to handle the integral that results from the integration by parts, including using a second integration by parts or another method on it.
- Recognize when an integration by parts will be needed even when it appears that there is only one term (let $v'(x) = 1$). (For example, integrating inverse functions such as $\ln x$ or $\arctan x$, or in other situations like integrating $e^{\sqrt{x}}$, when other methods don't apply.
- Be able to correctly handle limits of integration in an integration by parts.
- be able to recognize when rewriting a trig integral in terms of $\cos(x)$ and $\sin(x)$ will be helpful.
- Know the integrals of $\sec(x)$ and $\tan(x)$.
- Be able to evaluate integrals of the form $\frac{1}{x^2+a^2}$ and $\frac{1}{\sqrt{x^2-a^2}}$
- Be able to evaluate integrals of the form $\int \sin^m x \cos^n x dx$ (when at least one of n and m is odd, or when they are both even).
- Know when to use decomposition of partial fractions. Recognize when a long-division must precede the decomposition, and be able to do that calculation. Know the form of each partial fraction decomposition (linear factors, repeated linear factors, or irreducible quadratics). Be able to calculate the coefficients (your choice, you may calculate by substitution or by equating coefficients or a hybrid of these two methods).
- Recognize that any of the possible results of partial fraction decomposition cannot be simplified further by another partial fraction decomposition. Be able to integrate any of these possible outputs (such as $\frac{1}{2x+1}$, or $\frac{5}{(x-4)^2}$, or $\frac{4}{x^2+1}$ or $\frac{7}{x^2+9}$ or $\frac{3x}{x^2+1}$ or $\frac{x+7}{x^2+4}$ or $\frac{1}{x^2+4x+13}$ or $\frac{4x+1}{x^2+4x+13}$).
- Note that the second-to-last example above requires you to complete the square in the denominator, followed by a u/du sub, followed by a trig. substitution (or use a memorized formula). The last example requires you to complete the square, do a u/du sub, then split into two separate integrals, the first of which requires another u/du substitution, followed by a trig substitution.

- Know when and how to perform a trigonometric substitution. You are responsible for only the $x = a \tan(\theta)$ and the $x = a \sin(\theta)$ substitutions. You may use either drawings of triangles to help perform the substitution, or you may use the method in the book.
- After completing a trig substitution, an integral results that contains just trig functions. You should be able to evaluate these resulting integrals using the techniques listed above.
- You should recognize when you need to complete the square before starting a trig substitution. You should be able to complete the square, then do the necessary u/du substitution before or as part of the trig substitution.
- You should be able to estimate an integral using left- or right Riemann sums, or trapezoid or midpoint rules. You should be able to use the increasing/decreasing nature of a function and its concavity to compare the sizes of each of these estimations and the actual value of the integral.
- You should be able to recognize both types of improper integrals (an integral over an infinite region, or an integral with an infinite discontinuity at one of its endpoints or inside the interval). You should know how to rewrite these with a limit, and how evaluate the integral and then evaluate the limit to determine if the integral converges or diverges.

- For the improper integrals $\int_1^{\infty} \frac{1}{x^p} dx$ you should know which values of p give a convergent/divergent integral.
- Given an improper integral, you should be able to effectively and correctly compare it to a simpler integral that you can determine the convergence/divergence of, and use this comparison to determine if the original improper integral converges.
- Use integration to compute the area between two curves.
- Use integration to compute volumes of solids created by cross sections, revolutions by disk, and revolutions of cylindrical shells.