Math 2300, Midterm 2

March 9, 2015

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Albert Bronstein	8:00 - 8:50
☐	Section 002	Faan Tone Liu	9:00 - 9:50
☐	Section 003	Albert Bronstein	10:00 - 10:50
☐	Section 004	Joshua Frinak	11:00 - 11:50
☐	Section 005	Steven Weinell	11:00 - 11:50
☐	Section 006	Michael Roy	12:00 - 12:50
☐	Section 007	Clifford Blakestad	1:00 - 1:50
☐	Section 008	Krisztian Havasi	1:00 - 1:50
☐	Section 009	Pearce Washabaugh	2:00 - 2:50
☐	Section 010	Charles Scherer	3:00 - 3:50
☐	Section 011	Keli Parker	4:00 - 4:50
☐	Section 012	Andrew Healy	10:00 - 10:50
☐	Section 013	Jared Nishikawa	12:00 - 12:50
☐	Section 430R	Patrick Newberry	10:00 - 10:50

Question	Points	Score
1	4	
2	4	
3	8	
4	10	
5	20	
6	10	
7	10	
8	24	
9	10	
Total:	100	

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like ln(3)/2 as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (4 points) Which of the following integrals gives the arc length of the function $f(x) = 3\cos(x)$ from $x = 0$ to $x = \pi/4$? You do not need to show any work for this problem.

   (A) $\displaystyle\int_0^{\pi/4} \sqrt{1 + 9\cos^2(x)}\, dx$

   (C) $\displaystyle\int_0^{\pi/4} \sqrt{1 - 3\sin(x)}\, dx$

   (B) $\displaystyle\int_0^{\pi/4} \sqrt{1 + 9\sin^2(x)}\, dx$

   (D) $\displaystyle\int_0^{\pi/4} \sqrt{1 - 9\sin^2(x)}\, dx$

2. (4 points) Which of the following integrals gives the average value of the function $g(x) = x\ln(x)$ between $x = 1$ and $x = 10$? You do not need to show any work for this problem.

   (A) $\displaystyle\int_1^{10} x^2 \ln(x)\, dx$

   (C) $\displaystyle\int_1^{10} \frac{x\ln(x)}{9}\, dx$

   (B) $\displaystyle\int_1^{10} \frac{x\ln(x)}{10}\, dx$

   (D) $\displaystyle\int_1^{10} x\ln(x)\, dx$

3. Consider the region bounded by the x-axis, the y-axis, the line $x = 5$, and the curve $y = xe^{-x}$.

   (a) (4 points) Which of the following expressions gives the x-coordinate of the center of mass of the region? You do not need to show any work for this problem.

   (A) $\dfrac{\int_0^5 xe^{-x}\, dx}{\int_0^5 e^{-x}\, dx}$

   (C) $\dfrac{\int_0^5 x^2e^{-x}\, dx}{\int_0^5 xe^{-x}\, dx}$

   (B) $\dfrac{\int_0^5 x^2e^{-2x}\, dx}{\int_0^5 2xe^{-x}\, dx}$

   (D) $\dfrac{\int_0^5 xe^{-x}\, dx}{\int_0^5 x^2e^{-x}\, dx}$

   (b) (4 points) Which of the following expressions gives the y-coordinate of the center of mass of the region? You do not need to show any work for this problem.

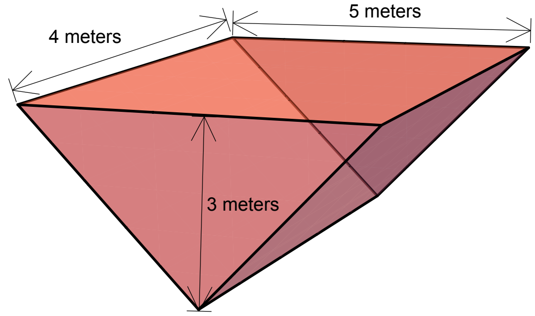
   (A) $\dfrac{\int_0^5 x^2e^{-2x}\, dx}{\int_0^5 2xe^{-x}\, dx}$

   (C) $\dfrac{\int_0^5 xe^{-2x}\, dx}{\int_0^5 xe^{-x}\, dx}$

   (B) $\dfrac{\int_0^5 x^2e^{-x}\, dx}{\int_0^5 xe^{-x}\, dx}$

   (D) $\dfrac{\int_0^5 x^2e^{-x}\, dx}{2\int_0^5 xe^{-x}\, dx}$

4. (10 points) The solid pictured below shows a tank filled with water. Set up an integral that represents the work required to pump all of the water out of the the top of the tank. Use ρ for density of water and g for the acceleration due to gravity. Show any work that you use to arrive at your answer. You do not need to evaluate the integral.



$$\int_{\square}^{\square} \boxed{\qquad\qquad\qquad\qquad} \, d\,\square$$

5. Determine if each of the following series **converges conditionally, converges absolutely, or diverges**. Recall that a series is conditionally convergent if it converges, but does not converge absolutely. Show all your work and carefully and fully justify your reasoning, including naming the convergence test(s) you are using.

   (a) (10 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$

(b) (10 points) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

6. (10 points) Determine if the following series converges or diverges. Show all your work and carefully justify your reasoning. If the series converges, give the value of its sum. $\sum_{n=1}^{\infty} \frac{3 \cdot 2^{n+1}}{5^{n-1}}$.

7. (10 points) The infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

converges to some real number s. We want to estimate s by computing the partial sum

$$s_n = \sum_{k=1}^{n} \frac{(-1)^k}{k^2} = -1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \cdots + \frac{(-1)^n}{n^2}$$

How many terms must the partial sum contain if we want to guarantee that the error is less than or equal to $.0001 = 10^{-4}$?

8. (24 points) For each of the following series, determine if it converges absolutely, converges conditionally or diverges. Recall that a series is conditionally convergent if it converges, but does not converge absolutely. You do not need to show any of your work.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(b) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{n^6}}$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan\left(n^2\right)}{n}$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(-\pi)^n}$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(f) $\displaystyle\sum_{n=1}^{\infty} \cos(\pi n)$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{(-n)^7}{2^n}$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{5n+n^3}}{2n^2+3}$

(A) converges absolutely

(B) converges conditionally

(C) diverges

9. (10 points) Select the best method for determining whether the following series converge or diverge. You do not need to show any of your work.

(a) $\sum \dfrac{10}{n\ln(n)}$

(A) alternating series test
(B) divergence test
(C) integral test
(D) ratio test

(b) $\sum \dfrac{3^{2n-1}}{(2n+1)!}$

(A) p-series
(B) divergence test
(C) geometric series
(D) ratio test

(c) $\sum \dfrac{5n^3}{\sqrt{n^7-4}}$

(A) alternating series test
(B) divergence test
(C) ratio test
(D) limit comparison test

(d) $\sum \dfrac{(-1)^n\sqrt{2n^2+3}}{n-3}$

(A) alternating series test
(B) divergence test
(C) integral test
(D) ratio test

(e) $\sum n^{-1/2}$

(A) alternating series test
(B) divergence test
(C) p-series
(D) ratio test