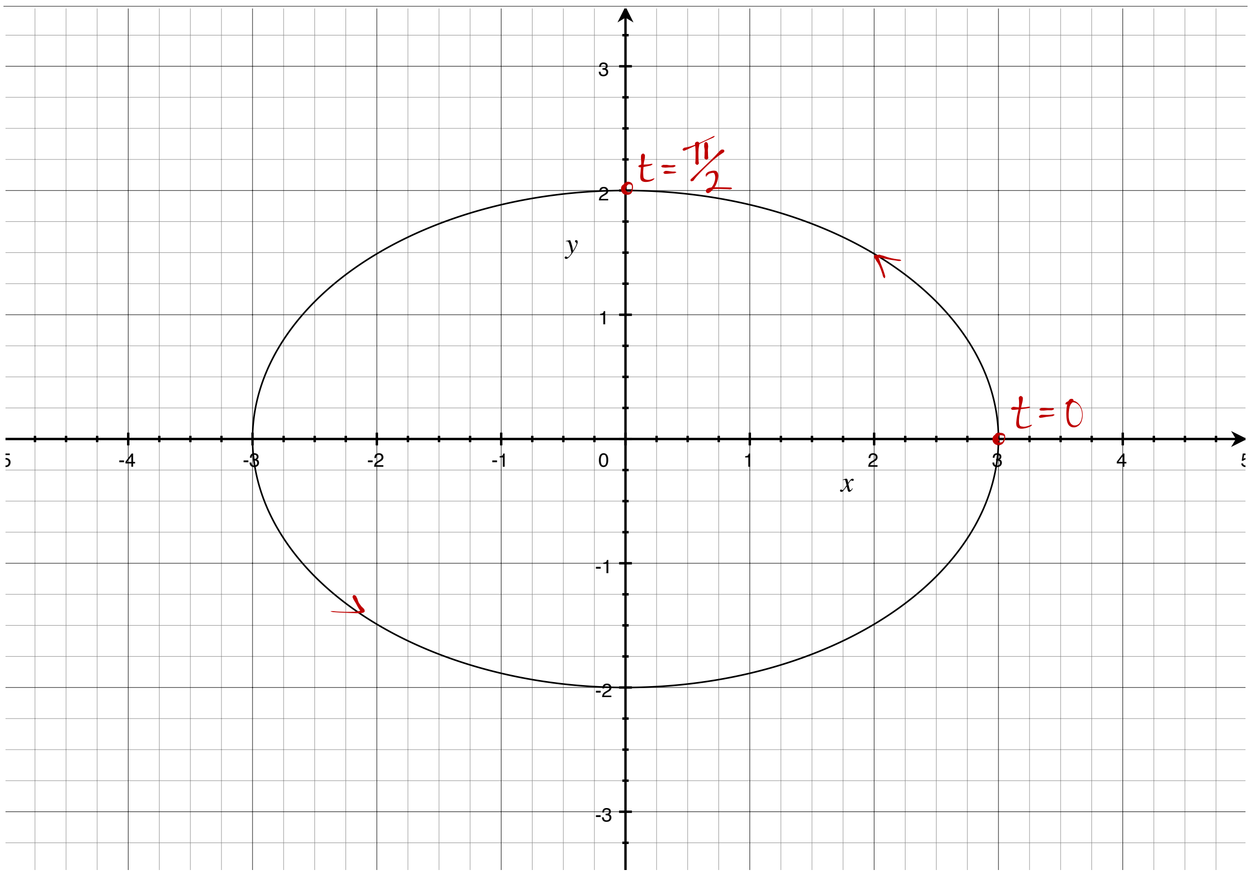

Figure 1: 10 (a)

counterclockwise

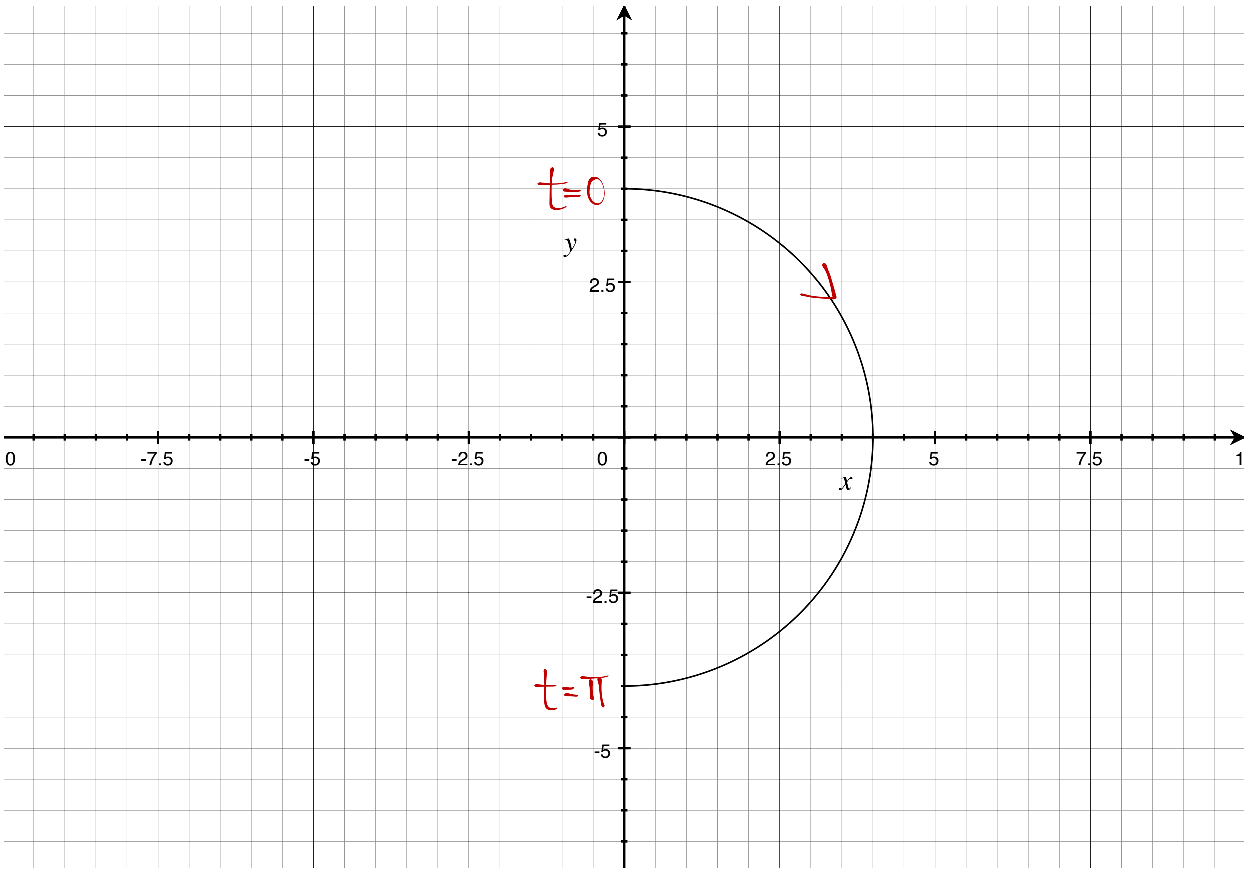

Figure 2: 10 (b)

clockwise

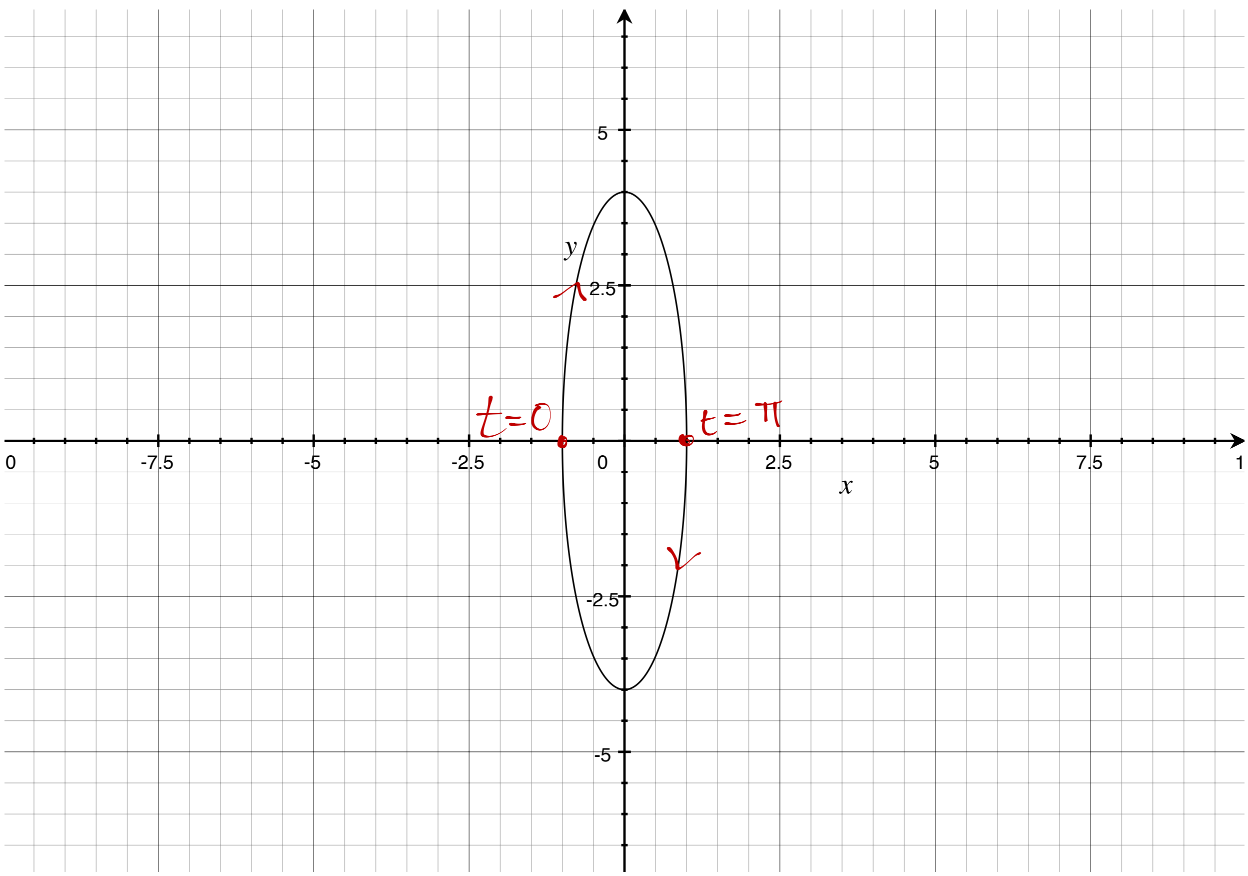

Figure 3: 10 (c) clockwise

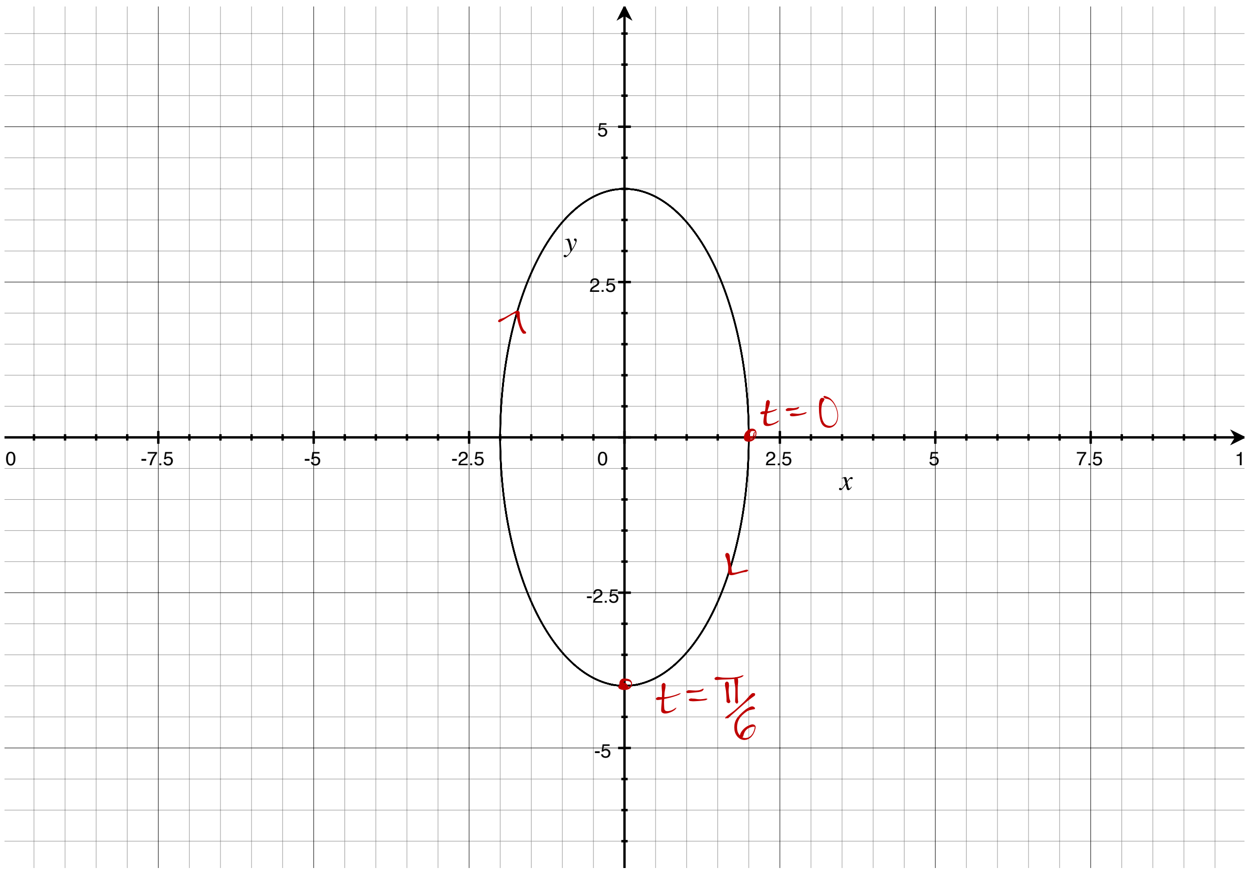

Figure 4: 10 (d)

Clockwise, goes around
3 times

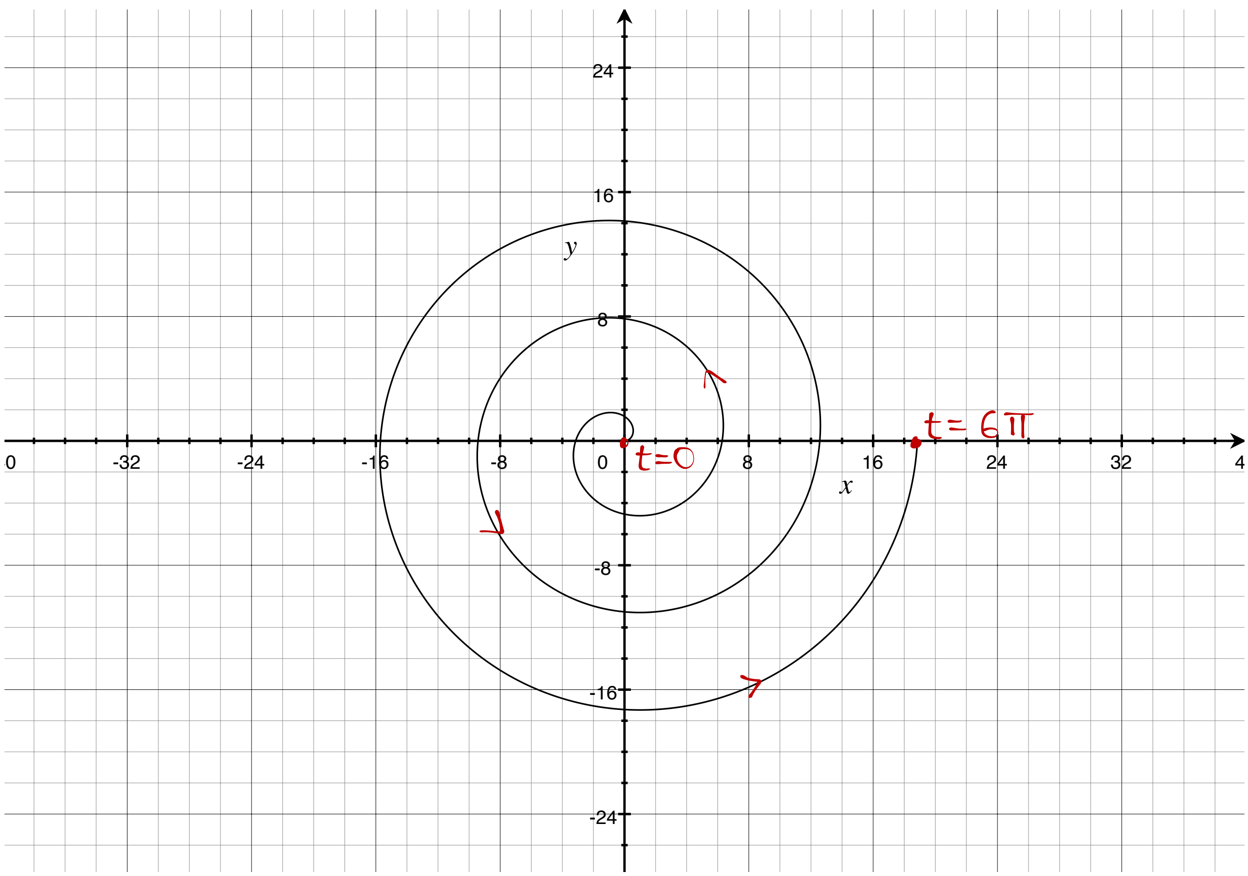

Figure 5: 10 (e)

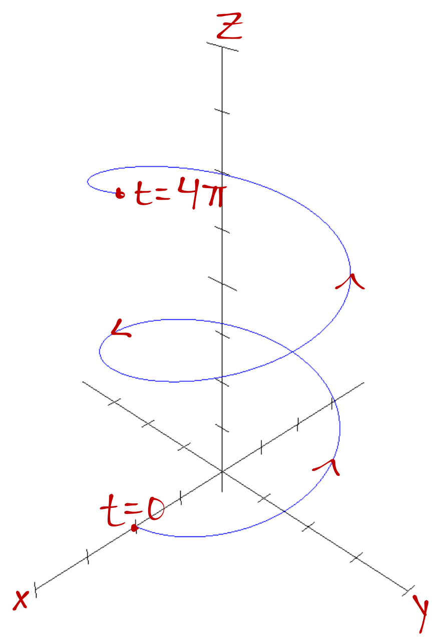

Figure 6: 10 (f)

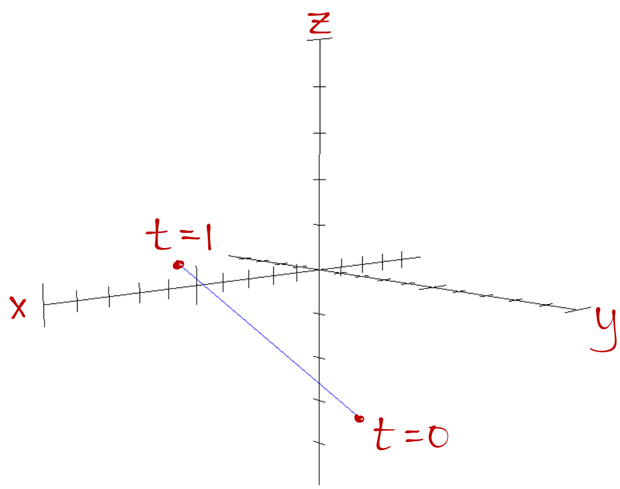

Figure 7: 10 (g)

line comes out of page from $t=0$ to $t=1$

11.) a) $x = 3\cos t$ $\quad 0 \le t \le 2\pi$
$y = -3\sin t$

b) $x = 3\sin t$ $\quad 0 \le t \le 2\pi$
$y = 5\cos t$ $\quad$ this will be clockwise

c) $x = 2 - 5t$ $\quad 0 \le t \le 1$ $\quad$ From the general form of $x = x_0 + (x_1 - x_0)t$
$y = 3 + 2t$ $\quad y = y_0 + (y_1 - y_0)t$

12.) For $x = t^3 - 3t$, $y = t^2 - 2t$
we calculate:

$$\frac{dy}{dt} = 2t - 2 \qquad \frac{dx}{dt} = 3t^2 - 3 \qquad \text{slope } \frac{dy}{dx} = \frac{2t-2}{3t^2-3} = \frac{2(t-1)}{3(t+1)(t-1)} = \frac{2}{3(t+1)}$$

so:

a) tangent line at $t = -2$ is

$$y = \frac{2}{3(-2+1)}\left(x - x(-2)\right) + y(-2)$$

$$y = \frac{2}{-3}(x+2) + 8$$

b) $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} \qquad \frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{2}{3(t+1)}\right) = \frac{-2}{3(t+1)^2}$$

so

$$\frac{d^2y}{dx^2} = \frac{\frac{-2}{3(t+1)^2}}{3t^2-3} = \frac{-2}{9(t+1)^3(t-1)}$$

at $t = -2$, we get

$$\boxed{\frac{-2}{27}}$$

c) speed at $t = -2$:

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{9^2 + (-6)^2} = \sqrt{9(9+4)} = \boxed{3\sqrt{13}}$$

d) From the slope $\frac{dy}{dx} = \frac{2}{3t+3}$, the tangent line is vertical at $t = -1$

e) The tangent line is never horizontal, $\frac{2}{3t+3} = 0$ has no solutions.

f) speed in general will be zero if $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are <u>both</u> zero at some t.

$2t - 2 = 0$ for $t = 1$
$3t^2 - 3 = 0$ for $t = 1$

so yes, the particle stops at $t = 1$.

g) Arclength is $\displaystyle\int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$, so

$$\int_{-2}^{1} \sqrt{(3t^2-3)^2 + (2t-2)^2}\, dt$$

h) Average speed will be the average value of the function

$\sqrt{(3t^2-3)^2 + (2t-2)^2}$, which can be computed as

$$\frac{1}{1-(-2)} \int_{-2}^{1} \sqrt{(3t^2-3)^2 + (2t-2)^2}\, dt \approx 12.639$$

note this integral is computing:

$$\frac{\text{distance travelled}}{\Delta \text{ in time}} = \text{average speed.}$$