SAMPLE MIDTERM II
EUCLIDEAN AND NON-EUCLIDEAN GEOMETRY

MATH 3210

Friday February 21, 2014

Name | ______________________________

Please answer all of the questions, and show your work.
All solutions must be explained clearly to receive credit.

1	2	3	4	5	
10	10	10	10	10	Total

Date: March 21, 2014.

1
10 points

1.(a). Define $\mathbb{A}^2_{\mathbb{R}}$ and $\mathbb{P}^2_{\mathbb{R}}$.

1.(b). Show that $\mathbb{P}^2_{\mathbb{R}}$ is isomorphic to the projective completion of $\mathbb{A}^2_{\mathbb{R}}$.

2
10 points

2. List the axioms of congruence.

3
10 points

3. Let $\mathbb{B}$ be a betweenness plane. Suppose that $A * B * D$ and $A * C * D$. Show using only the axioms of a betweenness plane, and properties of incidence planes, that if $B \neq C$, then either $A * B * C$ or $A * C * B$.

4
10 points

4.(a). In a Hilbert plane, show that if a ray emanates from an interior point of a triangle, then it intersects one of the sides of the triangle.

4.(b). In a Hilbert plane, show that a line cannot be contained in the interior of a triangle.

5. True or false.

5
10 points

5.(a) Suppose that A, B, C, D are points in a Hilbert plane, all lying on a common line. If $A * C * B$ and $A * B * D$, then $A * C * D$.

5.(b) Aristotle's continuity principle implies Archimedes' continuity principle.

5.(c) All Euclidean planes are isomorphic.

5.(d) Every projective plane is a Hilbert plane.

5.(e) Given an affine plane $\mathbb{A}$, there is a projective plane $\mathbb{P}$ with $\mathbb{A}$ isomorphic to a sub-plane of $\mathbb{P}$.

5.(f) In a betweenness plane $\mathbb{B}$, if L is a line and P is a point, then there exists a line through P perpendicular to L.

5.(g) The set of points of a Hilbert plane is infinite.

5.(h) In a Hilbert plane, if $\overrightarrow{AD}$ is between $\overrightarrow{AC}$ and $\overrightarrow{AB}$, then $\overrightarrow{AD}$ intersects segment BC.

5.(i) Let $\mathbb{P}$ be a finite projective plane. Assume there exists a line of $\mathbb{P}$ with exactly $n + 1$ points lying on it. Then every line of $\mathbb{P}$ has exactly $n + 1$ points lying on it.

5.(j) Suppose that $f : \mathbb{I} \to \mathbb{J}$ is a morphism of incidence planes. Then f is an isomorphism if and only if there exists a morphism $g : \mathbb{J} \to \mathbb{I}$ such that $g \circ f = Id_{\mathbb{I}}$.