SAMPLE FINAL EXAM
EUCLIDEAN AND NON-EUCLIDEAN GEOMETRY

MATH 3210

Monday May 5, 2014
7:30 AM - 10:00 AM
ECCR 151

Name ____________________________________

Please answer all of the questions, and show your work.
All solutions must be explained clearly to receive credit.

1	2	3	4	5	6	7	8	
10	10	10	10	10	10	10	10	Total

Date: April 29, 2014.

1
10 points

1.(a). Define a betweenness plane.

1.(b). Show that lying on the same side of a line defines an equivalence relation for points not on the line. I.e., in a betweenness plane, fix a line ℓ and let S be the set of points not lying on ℓ. For all $P, Q \in S$, define $P \sim Q$ if and only if P and Q are on the same side of ℓ. Show that $\sim$ defines an equivalence relation on S.

1.(c). Show using the definition of a betweenness plane that S has exactly two equivalence classes.

1.(d). Fix a betweenness plane. Given parallel lines ℓ and m and points A and B that lie on the opposite side of m from ℓ; i.e., for any point P on ℓ, A and P are on opposite sides of m, and B and P are on opposite sides of m. Prove that A and B lie on the same side of ℓ.

2
10 points

2. In this problem we work in a real Euclidean plane. Give a straight edge and compass construction of products and ratios of real numbers. In other words, fix a segment and set that to have length 1 so that we can define lengths in our plane. Then given two segments of length a and b respectively, construct two new segments, one of length ab and the other of length a/b. *You must prove that your construction works.*

3
10 points

In this problem we work in a Hilbert plane satisfying Dedekind's axiom.

3.(a). Define the defect δ of a triangle ΔABC.

3.(b). Show the defect is additive: Given a triangle ΔABC, if D is any point between A and B (see the figure), then $\delta(ABC) = \delta(ACD) + \delta(BCD)$.

4
10 points

4. In a Hilbert plane, show that opposite sides of a rectangle are congruent to each other.

5
10 points

In this problem, we work in a Hilbert plane satisfying Dedekind's axiom.

5.(a). What is the definition of a limiting parallel ray?

5.(b). What is the definition of a hyperbolic plane?

5.(c). Show that a hyperbolic plane is not Euclidean?

6
10 points

6.(a). Define the set of points in the Poincaré model.

6.(b). Define the set of lines in the Poincaré model.

6.(c). Define the incidence relation.

6.(d). State the incidence axioms.

6.(e). Rewrite the incidence axioms for the Poincaré model in terms of conditions on circles and lines in the real Euclidean plane.

7
10 points

7. In this problem, we work in a Hilbert plane satisfying Dedekind's axiom. Let ℓ be a line and let P be a point not on ℓ. Let $\overrightarrow{PY}$ be a limiting parallel ray to ℓ, and let X be a point on this ray between P and Y. Show that $\overrightarrow{XY}$ is a limiting parallel ray to ℓ through X.

8
10 points

8. True or false.

☐ **8.(a)** Let p, q, r be statements. Then the statement

$$[p \implies q] \iff [(p \wedge \sim q) \implies (r \wedge \sim r)]$$

may be true or false depending on whether p, q, r are true or false.

☐ **8.(b)** Given an (arbitrary) angle in a real Euclidean plane, one can trisect the angle with a straight edge and compass.

☐ **8.(c)** In a Hilbert plane, suppose that $A * C * B$ and $A * D * B$. Then $A * C * D$.

☐ **8.(d)** Every projective plane is a Hilbert plane.

☐ **8.(e)** There exists a real hyperbolic plane that is not equal to the Poincaré model.

☐ **8.(f)** There exists a Hilbert plane satisfying Aristotles's axiom such that the angle sum of a triangle is greater than 180°.

☐ **8.(g)** A Hilbert plane that admits a rectangle is semi-Euclidean.

☐ **8.(h)** Given a Hilbert plane satisfying Dedekind's axiom, if one Saccheri quadrilateral has summit angles of 90°, then so do all Sachheri quadrilaterals.

☐ **8.(i)** The summit angles of a Lambert quadrilateral are congruent.

☐ **8.(j)** In a Hilbert plane with no rectangles, given a line L and a point P not on L there are an infinite number of lines through P that are parallel to L.