Analytic Geometry and Calculus 2
MATH 2300

Wednesday April 11, 2012

Sample Midterm 3

THIS IS ONLY FOR REVIEW – THERE IS NO THIRD MIDTERM

On my honor as a University of Colorado at Boulder student I have neither given nor received unauthorized assistance on this exam.

Name:_____

DO NOT OPEN THIS EXAM UNTIL INSTRUCTED TO DO SO!

- ◯ **001** MARTINEZ ..... (8AM)
- ◯ **002** SPINA ..... (9AM)
- ◯ **003** ROSENBAUM ..... (10AM)
- ◯ **004** SHANNON ..... (11AM)
- ◯ **005** CASALAINA-MARTIN .... (11AM)
- ◯ **006** SCHERER ..... (12PM)
- ◯ **007** DAVISON ..... (1PM)
- ◯ **008** WAYNE ..... (1PM)

You may NOT use: books, notes, or calculators.

You SHOULD use: complete sentences and clear handwriting.

In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show all of your work, and give clear explanations.

DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	20 pts	
2	20 pts	
3	20 pts	
4	20 pts	
5	20 pts	
TOTAL	100 pts	

1
20 points

1. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

1.(a). $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

1.(b). $\displaystyle\sum_{n=0}^{\infty} \frac{1}{3^n}$

1.(c). $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{2^n}{n^2+1}$

1.(d). $\sum_{n=1}^{\infty}\frac{n!}{n^n}$

2
20 points

2. Find the **radius of convergence** and the **interval of convergence** for the power series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}.$$

[Hint: remember to check the endpoints.]

3
20 points

3. Consider the function $f(x) = \dfrac{2}{(1-x)^3}$.

3.(a). Find the degree 2 Taylor polynomial for $f(x)$ centered at 0. [Hint: $f(x) = 2(1-x)^{-3}$.]

3.(b). Find the Taylor series for $f(x)$ centered at 0.

4
20 points

4. Consider the function $f(x) = e^{2x}$.

4.(a). [10 points] Find the degree 3 Taylor polynomial for $f(x) = e^{2x}$ centered at 0.

4.(b). [2 points] Use the fact that $e^{1/5} < 3/2$ to show that

$$\frac{(2^4 e^{1/5})\left(\frac{1}{10}\right)^4}{4!} < 10^{-4}.$$

[This will be useful in (c).]

4.(c). [8 points] Let $P_3(x)$ be the degree 3 Taylor polynomial for $f(x) = e^{2x}$ centered at 0. Use (b) to show that for all $-\frac{1}{10} \leq x \leq \frac{1}{10}$,

$$|f(x) - P_3(x)| < 10^{-4}.$$

[Hint: the left hand side of the inequality in (b) may show up in a standard error estimation.]

5
20 points

5. Answer the following problems on differential equations.

5.(a). Is $y = \sin 2x$ a solution to the differential equation $y'' - 4y = 0$? Explain your answer.

5.(b). Find a solution to the differential equation $\dfrac{dy}{dx} = \dfrac{x^2 + 1}{y}$, with $y(0) = 1$.