PRACTICE MIDTERM II

MATH 3140

Friday April 1, 2011.

Name | ______________________________

Please answer the all of the questions, and show your work.

1	2	3	4	5	
10	10	10	10	10	total

Date: March 26, 2011.

1
10 points

1. Let $R = \{a + bx : a, b \in \mathbb{C}\} \subseteq \mathbb{C}[x]$ be the set of polynomials of degree at most 1. Define addition and multiplication on R by

$$(a + bx) + (a' + b'x) = (a + a') + (b + b')x$$

and

$$(a + bx)(a' + b'x) = aa' + (ab' + a'b)x$$

for all $a, a', b, b' \in \mathbb{C}$. Show that $(R, +, \cdot)$ is a ring.

2
10 points

2. Recall that for a commutative ring R with unity $1 \neq 0$, we define $R[x]$ to be the ring of polynomials in x with coefficients in R. Consider the map

$$\phi : \mathbb{Z}[x] \to \mathbb{Z}_4[x] \quad \text{given by the rule} \quad \sum_{k=0}^{n} a_k x^k \mapsto \sum_{k=0}^{n} [a_k] x^k.$$

2(a) [6 points]. Show that ϕ is a homomorphism of rings.

2(b) [2 points]. Describe the kernel of ϕ (in terms of the coefficients of the polynomials).

2(c) [2 points]. Is ϕ surjective?

3
10 points

3. Let G be a group with center $Z(G)$. Assume that $G/Z(G)$ is cyclic.

3(a) [8 points]. Show that $Z(G) = G$. [Hint: Show there exists $g \in G$ such that for any $g_1 \in G$, there is a $z_1 \in Z(G)$ and $n_1 \in \mathbb{Z}$ such that $g_1 = g^{n_1} z_1$.]

3(b) [2 points]. Show that the commutator subgroup of G is trivial; i.e. $C(G) = \{e_G\}$.

4
10 points

4. Consider the dihedral group D_n, with $n \geq 3$. Recall the notation we have been using: D_n has identity element Id, and is generated by elements R and D, satisfying the relations $R^n = D^2 = Id$ and $RD = DR^{-1}$. Consider the cyclic subgroup $\langle R^2 \rangle$.

4(a) [6 points]. Show that $\langle R^2 \rangle$ is a normal subgroup of D_n.

4(b) [4 points]. Find the order of the group $D_n/\langle R^2 \rangle$ [Hint: this may depend on the parity of n.]

5
10 points

5. True or false. (Please provide a sentence or two of explanation.)

5(a). If G is a group of order n and k divides n, then G has a subgroup of order k.

5(b). The alternating group A_5 is simple.

5(c). The kernel of a homomorphism is a normal subgroup.

5(d). Every element in a ring has an additive inverse.

5(e). Let R be a ring with unity 1_R, and let $a \in R$. If $a^2 = a$, then $a = 0_R$ or $a = 1_R$.