MATH 232: HOMEWORK 5

DUE MONDAY APRIL 28

Problems

(1) Let X be a smooth projective variety, and H a very ample divisor on X. If E is a divisor on X such that $|E|$ is base point free, then show that $H + E$ is very ample.

(2) If $S \subset \mathbb{P}^n$, $n \geq 3$, is a smooth surface containing a line $\ell \subset \mathbb{P}^n$, and H is a hyperplane section, show that $|H - \ell|$ is base point free.

(3) Let Y be a smooth projective variety. Show that there is a one to one correspondence between étale double covers $X \to Y$, and line bundles $L \in \mathrm{Pic}^0(Y)$ such that $L \otimes L \cong \mathscr{O}_Y$.

(4) Let A be an abelian surface, with involution $\tau : A \to A$ given by $a \mapsto -a$.

  (a) Show that τ has 16 fixed points.

  (b) Let $\hat{A}$ be the blow-up of A at the 16 points. Show that τ induces an involution of $\hat{\tau}$ on $\hat{A}$, and that $\hat{A}/\hat{\tau}$ is a $K3$ surface.

  (c) Show that A/τ is a surface with 16 ordinary double points, and that there is a map from $\hat{A}/\hat{\tau} \to A/\tau$ making the following diagram commute.

$$\begin{array}{ccc} \hat{A} & \longrightarrow & \hat{A}/\hat{\tau} \\ \big\downarrow & & \big\downarrow \\ A & \longrightarrow & A/\tau \end{array}$$

(5) Suppose that $f : X \to Y$ is a morphism of a smooth projective variety to a smooth curve Y. Show that there is a commutative diagram

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ {\scriptstyle h}\big\downarrow & & \big\uparrow{\scriptstyle g} \\ C & = & C \end{array}$$

where C is smooth, h has connected fibers, and g is finite.

(6) Suppose X and Y are smooth projective varieties.

(a) Show that $\kappa(X \times Y) = \kappa(X) + \kappa(Y)$.
(b) If $f : X \to Y$ is a finite surjective morphism, show that $\kappa(X) \geq \kappa(Y)$. [Note that this also holds for generically finite surjective morphisms.]
(c) Give an example $f : X \to Y$ of a surjective morphism with $\dim X > \dim Y$ and $\kappa(X) < \kappa(Y)$.