Homework 9

Due Tuesday, December 19

Exercises

1. Let X be a smooth projective variety, and let $\mathcal{O}_X(1)$ be a very ample line bundle on X. Show that the hermitian form $H : H^{0,1}(X) \times H^{0,1}(X) \to \mathbb{C}$ given by

$$H(\phi, \psi) = -2i \int_X \phi \wedge \bar{\psi} \wedge c_1(\mathcal{O}_X(1))^{n-1}$$

   defines a polarization on $Pic^0(X)$.

2. Let X be a smooth threefold. A theorem of Clemens and Griffiths states that if X is rational, then (JX, Θ_X) is isomorphic to the product of Jacobians of curves. In this exercise, we show that the converse is false. Let C be a smooth curve of genus $g > 0$, and let $X = C \times \mathbb{P}^2$. Show that the intermediate Jacobian (JX, Θ_X) is isomorphic to (JC, Θ_C). On the other hand, find a smooth rational threefold Y such that $(JY, \Theta_Y) \cong (JC, \Theta_C)$.

3. Let $I \subset \mathcal{A}_5$ denote the locus of ppavs which are the intermediate Jacobian of a smooth cubic hypersurface in $\mathbb{P}^4$, and $\bar{I}$ its closure. For which curves C of genus five, is it possible that $JC \in \bar{I}$. It is a result of Collino's that all of these Jacobians do appear in the boundary.

4. Let X be an abelian surface, with Kummer variety $Kum(X)$. Let Y be the blow-up of $Kum(X)$ at the 16 singular points. Show that Y is a K3 surface, i.e. K_Y is trivial, and $H^0(Y, \Omega^1_Y) = 0$.

5. Suppose $(A, \Theta) \in \mathcal{A}_g$. Consider the morphism $A \xrightarrow{|2\Theta|} \mathbb{P}^{2^g - 1}$. Find the degree of the image. In particular, show that if (A, Θ) is a surface, then the image of A is a hypersurface of degree four in $\mathbb{P}^3$. Conclude that in this case, $Kum(X)$ admits a four dimensional family of tri-secants.