Homework 6

Due Friday, November 10

Exercises

1. Let X be an analytic space, and $\mathcal{F}$ a coherent sheaf on X. Given two presentations of $\mathcal{F}$,
$$\mathcal{O}_X^{n_1} \overset{\alpha_1}{\to} \mathcal{O}_X^{m_1} \to \mathcal{F} \to 0$$
and
$$\mathcal{O}_X^{n_2} \overset{\alpha_1}{\to} \mathcal{O}_X^{m_2} \to \mathcal{F} \to 0,$$
show that the $(m_1 - k)$-th degeneracy locus of α_1 is the same as the $(m_2 - k)$-th degeneracy locus of α_2.

2. Let $S \subseteq X$ be a smooth subvariety of an abelian variety, and let $f : S \times S \to X$ be the difference map $(s, s) \mapsto s - s$. Show that $f^{-1}(0)$ is smooth as a scheme.

3. As in the last homework assignment, let
$$\phi_d : C_d \times C_d \to Pic^0(C)$$
be the difference map, and let V_d be the image. Show that if C is not hyperelliptic, then the projectivized tangent cone cone to V_1 at $\mathcal{O} \in Pic^0(C)$ is the canonical curve.

4. More generally, show that if C does not have a g^1_d, then the projectivized tangent cone cone to V_d at $\mathcal{O} \in Pic^0(C)$ is the d-secant variety to the canonical curve.

5. Let C be a genus five curve. Show that if C is hyperelliptic, then W^1_3 is isomorphic to C, and W^1_4 is isomorphic to C_2. If C is not hyperelliptic, and has a g^1_3, show that W^1_3 is a point. Finally, in the case that C is not hyperelliptic, show that W^1_4 is one dimensional.

6. If C is a genus six curve, give a description of W^1_4.