Homework 2

Due Friday, October 13

Exercises

1. Let $X = V/\Lambda$ be a complex torus. Let $v \in V$ be a representative of $x \in X$, and let $L = L(H, \chi)$ be a line bundle on X. Show

$$t_x^* L(H, \chi) = L(H, \chi exp(2\pi i Im H(v, \cdot))).$$

2. Prove the theorem of the square: For $x, y \in X$ a complex torus,

$$t_{x+y}^* L \cong t_x^* L \otimes t_y^* L \otimes L^{-1}.$$

3. If $f : Y \to X$ is a homomorphism of complex tori, and L is a line bundle on X, show that the following diagram commutes:

$$\begin{tikzcd} X \arrow[r, "\phi_L"] & \hat{X} \arrow[d, "\hat{f}"'] \\ Y \arrow[u, "f"] \arrow[r, "\phi_{f^*L}"] & \hat{Y}. \end{tikzcd}$$

4. Suppose L_1 and L_2 are line bundles on a complex torus X of dimension g, and L_2 is positive definite. Show that L_1 is positive definite if and only if $L_1^i L_2^{g-i} > 0$ for all $0 \le i \le g$. [Hint: See BL p.76.]

5. For a nondegenerate line bundle L on a complex torus X (i.e. $c_1(L)$ is a nondegenerate Hermitian form), show that L is ample if and only if L is effective. [Hint: Use the theorem stated in class for the cohomology of line bundles on a complex torus.]